Supersymmetric Model-Building in the Era of LHC Data: From Struggles with Naturalness to the Simple Delights of Fine-Tuning

by

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Abstract

The Standard Model (SM) of particle physics has withstood decades of experimental tests, making it the crowning achievement of 20$^{\text{th}}$ century physics. However, it is not a complete description of nature. Observations have revealed that most of the matter in the universe is not of the baryonic form described in the SM but rather something else known as dark matter. The SM also has theoretical shortcomings: 1) No explanation for the widely-varying masses of different particles (flavor puzzle); 2) Failure of the couplings that characterize the strength of the three SM forces to unify at a high energy scale; 3) Instability of the Higgs mass (hierarchy problem).

The simplest version of supersymmetry (SUSY) introduces a partner for each SM particle, resulting in the Minimal Supersymmetric Standard Model (MSSM). The lightest of these is stable and an appealing dark matter candidate, and the extra particle content yields good gauge coupling unification. Most model-building, however, has been inspired by the natural solution that the MSSM provides to the hierarchy problem when the superpartner masses are close to the weak scale, leading to the paradigm of the Natural (weak-scale) MSSM. Although the first run of the Large

ABSTRACT

Hadron Collider (LHC) did not operate at the design energy, the data is already in tension with the idea of naturalness, as the bounds on some superpartner masses in vanilla models are significantly above the weak scale. We address this by constructing a hybrid of the two most appealing SUSY breaking mechanisms (gauge and anomaly mediation) that compresses part of the superpartner spectrum and reduces experimental sensitivity, thereby loosening the constraints.

Nonetheless, the recent discovery of a Higgs-like particle at the LHC with a mass ≈ 125 GeV that can only be obtained in the weak-scale MSSM with fairly heavy superpartners casts serious doubt on naturalness. It does, however, point in the direction of a different paradigm in the MSSM known as Split SUSY, where only the superpartners that are potential dark matter candidates are light. We present a simple realization of a modification of Split SUSY, called Mini-Split SUSY, where all of the superpartner masses are determined by just one parameter. We show that it easily accommodates the Higgs mass, preserves gauge coupling unification, and has a good dark matter candidate. We then exploit the defining features of the Mini-Split framework to obtain a radiative solution to the flavor puzzle, where the hierarchy of SM particle masses is explained by successive orders of quantum corrections.

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Finally, I extend my heartfelt gratitude to my family, knowing that without their love, unwavering support, and belief in me, this work would never have been completed.

Dedication

This thesis is dedicated to my parents, Andrzej and Elżbieta, who from a young age instilled in me a deep respect for education and made many sacrifices to ensure that I always had a nurturing environment in which to learn and grow.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 The Standard Model

The Standard Model (SM) of particle physics is a description of nature at what is currently understood to be the most fundamental level, containing a collection of elementary particles and their interactions. It is formulated as a quantum field theory (QFT), which is the fully self-consistent marriage of quantum mechanics and special relativity. In this theory, each type of particle is the quantum excitation of a distinct, dynamical field that pervades all of spacetime, and these particles have local, point-like interactions that are specified in an object called a Lagrangian, denoted $\mathcal{L}$ [1]. At the classical level, $\mathcal{L} = K - V$, where K is the kinetic energy and V the potential energy of the system. Roughly speaking, each quantum field can be described as

[1] Strictly speaking, this is a density, and the actual Lagrangian is the integral of this over space, $L = \int d^3x\, \mathcal{L}$. For the sake of brevity, we will refer to $\mathcal{L}$ simply as the Lagrangian.

a network of linked harmonic oscillators, one at each point in space. Because it is a quantum system, the energies of the oscillators are quantized, and, as already mentioned, these discrete packets of energy are the particles. The interactions in $\mathcal{L}$ therefore specify how the different types of oscillators (fields) are coupled. As energy is transferred from one type of oscillator to another, some particles are destroyed and new ones appear. Thus, at the quantum level, the terms in $\mathcal{L}$ are understood as operators that create/annihilate particles.

The Lagrangian controls the dynamics of the theory and contains all the terms that are allowed by the symmetries of the system that it describes. All relativistic QFTs are invariant under a set of continuous spacetime symmetries–translations, rotations, and boosts–collectively known as Poincaré invariance. It is a set of internal symmetries, known as gauge symmetries and associated with Lie groups, that define the SM. These gauge symmetries can be loosely described as rotations in field space. In a gauge theory, the forces between matter particles arise from the exchange of gauge bosons. These gauge bosons themselves are the excitations of corresponding gauge fields, which are needed to promote the gauge symmetry from a global one, where, for each field, the transformation is the same at every spacetime point, to a local one, i.e. a transformation with coordinate dependence.

Once one specifies the field (particle) content, the internal symmetries, and how the fields transform under them, the Lagrangian is fixed. One can then calculate the probabilities for various scattering processes in perturbation theory using Feynman diagrams. The numerical factors assigned to the individual pieces of the diagram, the so-called Feynman rules, are derived from $\mathcal{L}$ and represent for example the amplitude, called the propagator, for a particle to propagate from one spacetime point to another, or the amplitude for an interaction, known as a vertex factor.

The SM is the most general, renormalizable QFT invariant under an $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group. Every known particle falls into a representation of one of these groups. The SM field content and charge assignments are listed in Tables 1.1 and 1.2. As mentioned above, the fields can be categorized as either matter or gauge. The matter fields all correspond to spin-1/2 fermions and can be further classified as either quarks or leptons. Only two types of quarks, known as up and down, are needed to form the protons and neutrons that comprise atomic nuclei, while one type of lepton, the electron, is needed to complete the atom. The electron has associated with it another type of lepton, known as an electron neutrino. This structure is then repeated two more times, resulting in three generations of 1) quarks, $\{q^i_L = (u^i_L, d^i_L), u^i_R, d^i_R\}$, where u and d denote up and down type, respectively, and $i = 1, 2, 3$ labels the generation; and 2) leptons $\{l^i_L = (\nu^i_L, e^i_L), e^i_R\}$, where e denotes electron type and ν neutrino type. The different generations of quarks of the same type are usually referred to as different flavors. The gauge fields, $\{g^a_\mu, W^\alpha_\mu, B_\mu\}$, are spin-1 (vector) bosons and as the mediators of the $SU(3)_c, SU(2)_L, U(1)_Y$ interactions, respectively, are all charged as adjoints; for $SU(N)$, the dimensionality of the adjoint representation (number of generators), which the upper index on the label runs over,

Field	$i=1$	$i=2$	$i=3$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
q_L^i	$\binom{u_L}{d_L}$	$\binom{c_L}{s_L}$	$\binom{t_L}{b_L}$	$\mathbf{3}$	$\mathbf{2}$	$1/6$
u_R^i	u_R	c_R	t_R	$\mathbf{3}$	$\mathbf{1}$	$2/3$
d_R^i	d_R	s_R	b_R	$\mathbf{3}$	$\mathbf{1}$	$-1/3$
l_L^i	$\binom{\nu_{eL}}{e_L}$	$\binom{\nu_{\mu L}}{\mu_L}$	$\binom{\nu_{\tau L}}{\tau_L}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
e_R^i	e_R	μ_R	τ_R	$\mathbf{1}$	$\mathbf{1}$	-1

Table 1.1: The SM matter field content and respective charge assignments. Note that all fields have spin-1/2. The index $i = 1, 2, 3$ labels the generation.

is given by $N^2 - 1$. The gauge bosons of the $SU(3)_c$ and $SU(2)_L \times U(1)_Y$ subgroups do not mix and thus these can be considered separate sectors.

The Quantum Chromodynamics (QCD) sector describes the $SU(3)_c$ interactions of quarks through the strong force. The quarks carry one of three possible color charges–red, green, or blue (hence the c in the gauge group subscript)–and the gluons g that are exchanged carry both color and anti-color charges. However, it is not possible to observe an isolated quark because the strong force increases in strength as the separation distance between two color-charged objects is increased. Thus, we can only see color-neutral bound states of quarks, called hadrons, the most well-known of which are the protons and neutrons that are found in an atomic nucleus. As the $SU(3)_c$ symmetry is a symmetry of the quantum vacuum, the gluons are massless.

The electroweak sector is the remaining $SU(2)_L \times U(1)_Y$ gauge symmetry. Like

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	spin
H	$\mathbf{1}$	$\mathbf{2}$	1/2	0
g_μ^a	$\mathbf{8}$	$\mathbf{1}$	0	1
W_μ^α	$\mathbf{1}$	$\mathbf{3}$	0	1
B_μ	$\mathbf{1}$	$\mathbf{1}$	0	1

Table 1.2: The SM Higgs and gauge field content and respective charge assignments.

$SU(3)_c$, $SU(2)_L$ is a non-Abelian group, so the corresponding gauge bosons W^α are also charged under it. It is different, however, because it is chiral; the "L" subscript indicates that only the left-handed projections of fermion fields participate in these interactions. These are grouped into the $SU(2)$ doublets q_L^i for quarks and l_L^i for leptons. While we have only observed one massless gauge boson, the photon, the electroweak sector initially has 4 massless gauge bosons, $\{W^+, W^-, W^3, B\}$. The charged gauge bosons $W^\pm$ are involved in beta decay, among other processes. Introducing a mass term for a gauge field by hand breaks the gauge symmetry, so giving mass to the gauge bosons must be done in a different way. This is accomplished in the SM through spontaneous symmetry breaking (SSB). In this mechanism, the equations of the theory are invariant under the full symmetry group, thus preserving the nice properties of gauge theories; however, part of the symmetry is "broken" by the low-temperature vacuum, which is only invariant under some subgroup. The electroweak symmetry breaking (EWSB) $SU(2)_L \times U(1)_Y \to U(1)_{EM}$ leaves only one unbroken generator, whose corresponding massless force carrier is the photon A_μ that is formed from a linear combination of the electrically neutral gauge bosons of the electroweak group, i.e. W^3 and B.

Electroweak symmetry breaking is accomplished by introducing what is the only fundamental spinless (scalar) particle in the SM, the Higgs boson, H. Since the Higgs is charged as a fundamental under $SU(2)$, it is a doublet of complex scalar fields, $H = (H^+, H^0)$, with four real degrees of freedom. It acquires a vacuum expectation value (VEV) in the real part of the neutral component, $\langle H \rangle = (0, v)$, giving masses to the charged W bosons and the Z boson (the linear combination of W^3 and B orthogonal to the photon). The measured values of the W and Z masses fix $v = 174$ GeV, defining the weak scale. The VEV is determined by minimizing the Higgs potential:

$$V_H = -\mu^2 |H|^2 + \frac{\lambda}{2} |H|^4, \qquad \mu^2 > 0,\, \lambda > 0, \tag{1.1}$$

yielding

$$v = \sqrt{\mu^2/\lambda}\,. \tag{1.2}$$

As a massive gauge boson has an additional longitudinal polarization, each of the gauge fields "eats" one of the other three Higgs components to acquire this extra degree of freedom. This leaves only one dynamical real scalar, h, corresponding to excitations above the VEV: $h \equiv \sqrt{2}(Re(H^0) - v)$ (the coefficient is needed so that h has canonical normalization for a real scalar field), and this is what one usually refers to as simply "the Higgs." Making this replacement in Eq. (1.1), we find its mass to be $m_h^2 = 2\lambda v^2$. Although the Higgs mechanism was postulated in the 1960's, a Higgs-like particle was discovered only very recently in 2012 at the Large Hadron Collider (LHC) [4, 5]. The analyses conducted since then using the additional data collected at the LHC indicate that its properties are very similar to that of the spin-0 boson in the SM Higgs mechanism, suggesting that it indeed is the Higgs [6–8].

The interaction of a fermion with a gauge field can be schematically represented (ignoring group theory structure) as the Lagrangian term $g\overline{\psi}\gamma^\mu\psi X_\mu$, where ψ is the fermion, X_μ is the gauge field, and g is a dimensionless parameter known as the gauge coupling, which characterizes the strength of the interaction. In Quantum Electrodynamics (QED), X_μ is the photon and g is just the electric charge e. However, g is not a constant but depends on the length scale characterizing the process, or equivalently (in natural units) the inverse energy scale. This is because the quantum vacuum is not empty but contains virtual particle-antiparticle pairs, whose existence is constrained by the energy-time uncertainty principle. Formally, these particle-antiparticle pairs appear in the loop corrections to gauge boson propagators in Feynman diagrams. We can take QED as an example to understand the physical effect of such pairs. The virtual positrons will screen a target electron, polarizing the vacuum and reducing the strength of the interaction. The charge constant familiar from electrostatics is actually the fully-screened charge, i.e. the charge seen as the distance between two electrons is taken to infinity, or equivalently, as the energy of the scattering process tends to zero. As the energy is increased, there is more penetration through the positron cloud and the effective charge increases. In non-Abelian theories, the gauge bosons themselves are charged and so also contribute to the polarization, but it turns out that they produce an anti-screening effect. Thus, for such theories, the evolution of the coupling with energy scale, known as running, depends on the number of charged fermions and gauge bosons. Like e, the gauge coupling g_2 of $SU(2)_L$ increases with increasing energy, whereas the coupling g_3 of QCD shows the opposite behavior, with the strength of the interaction growing with increasing distance, explaining why we only observe hadrons. In general, the running of the coupling is governed by the renormalization group equation (RGE):

$$\frac{dg}{dt} = \beta(g) \qquad t = \log(Q/Q_0) \tag{1.3}$$

where Q is the renormalization scale, which in a perturbative calculation should be taken close to the energy characterizing the process in question, and Q_0 is the scale at which the boundary condition is specified.

The running of the gauge couplings is just one example of the scale dependence of operator coefficients described by the renormalization group. Roughly speaking, one can construct an effective theory valid below some energy scale E by "integrating out" the degrees of freedom with larger momentum. More precisely, such an effective theory is described by $\mathcal{L}_{eff} = \mathcal{L} + \Delta\mathcal{L}$, where $\Delta\mathcal{L}$ is essentially obtained by evaluating all the loop diagrams (at a certain order in perturbation theory) arising from $\mathcal{L}$ with loop momentum restricted to the range $E < |k| < \Lambda$ (the momentum slice that is being removed), where Λ is the scale up to which $\mathcal{L}$ itself is valid. Hence, in general this procedure corrects the operator coefficients, although new operators not present in the original $\mathcal{L}$ may be generated in the process. Loosely identifying the Q above with E, this provides an intuitive way to understand renormalization group evolution. Furthermore, in processes with energies $< E$ where the effective theory is applicable, particles with masses greater than this scale are not kinematically accessible, so they can be eliminated from the theory. They can be "integrated out" completely at the scale E, with all of their virtual effects encoded in additional "threshold" corrections to operators.

In any theory with a scalar, it is usually possible to write additional interaction terms, known as Yukawa interactions, between the scalar and the fermions consistent with all the symmetries. In the SM, these take the form

$$\mathcal{L}_{yuk} = -y^u_{ij}\overline{q}^i_L \tilde{H} u^j_R - y^d_{ij}\overline{q}^i_L H d^j_R - y^e_{ij}\overline{l}^i_L H e^j_R + h.c., \tag{1.4}$$

where the $y^{u,d,e}_{ij}$ are dimensionless parameters known as Yukawa couplings and $\tilde{H} \equiv i\sigma^2 H^*$ is the charge conjugate of H. Replacing the Higgs by its VEV, we see that this results in Dirac mass terms for the fermions. For example, considering just the quarks, we have

$$\mathcal{L}_{yuk} \supset -v\, y^u_{ij}\overline{u}^i_L u^j_R - v\, y^d_{ij}\overline{d}^i_L d^j_R + h.c., \tag{1.5}$$

so that we can identify the mass matrices as $m^{u,d}_{ij} \equiv v\, y^{u,d}_{ij}$. We need to then diagonalize the mass matrices by performing unitary transformations on the fields, thereby switching to the physical (mass) basis:

$$u_L^{(g)} = V_u\, u_L^{(m)}, \qquad u_R^{(g)} = U_u\, u_R^{(m)} \tag{1.6}$$

$$d_L^{(g)} = V_d\, d_L^{(m)}, \qquad d_R^{(g)} = U_d\, d_R^{(m)}, \tag{1.7}$$

where g indicates the original (gauge) basis, and m the mass basis in which the mass matrices are diagonal. After diagonalization, the coupling of the quark mass eigenstates to the dynamical Higgs h is given by $y_i = m_i/v$. Since we have to perform separate rotations on u_L and d_L, the terms that mix u_L and d_L, i.e. those involving $W^\pm$, which are diagonal in the gauge basis, acquire flavor off-diagonal contributions after the rotation to mass basis. For example,

$$\mathcal{L} \supset \overline{u}_L V_{CKM}\, d_L\, W^+, \tag{1.8}$$

where $V_{CKM} = V_u^\dagger V_d$ is the Cabbibo-Kobayashi-Maskawa (CKM) matrix and it is understood that from now on all quark fields are in the mass basis. As a 3×3 unitary matrix, the CKM matrix can be described by 4 independent real parameters, consisting of three angles and one phase, which have been measured precisely by experiment [9] and are presented in Fig. 4.12. The SM fermion masses are listed in Table 1.3.

Although it is in remarkable agreement with experiment, the SM is still incomplete, failing to describe a number of phenomena. First, it does not include gravity, so it is at best an effective theory valid only up to the scale $M_{Pl} \approx 10^{19}$ GeV at which gravitational effects become important. Second, the particles that it describes

Particle	Mass (GeV)
u	1.38×10^{-3}
d	2.82×10^{-3}
c	0.638
s	5.70×10^{-2}
t	172.1
b	2.86
e	0.487×10^{-3}
μ	0.103
τ	1.75

Table 1.3: SM fermion masses, taken from [1]. These are actually the running masses at M_Z and not the pole (physical) masses, because the pole masses of the light quarks are not well defined. Although known to be small but non-zero, neutrino masses are not present in the SM.

comprise only about 5 percent of the total energy density of the universe. About 25 percent of what is missing is dark matter, so named because it does not interact electromagnetically and hence cannot be seen, only detected through its gravitational pull (the rest is attributed to dark energy). It is widely believed that dark matter is made up of a new type(s) of particle(s). This is because 1) Big Bang Nucleosynthesis (BBN), the theory that explains how heavier elements formed in the early universe, places a strong upper bound on the amount of baryonic dark matter, i.e. neutral objects made out of ordinary (baryonic) matter; and 2) This explanation is most consistent with the observed large-scale structure in the universe. Based on the dark matter abundance today, this particle is most commonly assumed to be a weakly-interacting massive particle (WIMP), i.e. a particle charged under the weak force with no strong interactions and massive enough to be non-relativistic at a certain time in the early universe.

Third, the SM does not explain why the different generations of fermions have such widely varying masses. Naturally one would expect all of the Yukawa couplings to be roughly $\mathcal{O}(1)$, which would imply masses of the same order for all flavors. Next, although it unifies the electromagnetic and weak interactions, it says nothing about how they relate to the strong interaction. Given the success of the electroweak model and the appeal of simple unified theories, there is good theoretical motivation for grand unified theories (GUT), where the SM is a subgroup of a larger simple group. It turns out that the smallest possible group that the SM fits into is $SU(5)$.

However, with just the SM field content, one finds, upon running the gauge couplings as described above, that they come close to but do not quite meet at a very high energy scale $M_{GUT} \approx 10^{16}$ GeV.

Finally, there is a problem associated with the instability of the weak scale to quantum corrections that want to drive the Higgs VEV v up to the Planck scale, which has its origin in an issue with the Higgs mass. Returning to Eq. (1.1), we see that in order to get the right value for the VEV (we reiterate that this is fixed by the experimental values of the W and Z boson masses), we need μ to be of order the weak scale itself, since λ must be perturbative, i.e. $\mathcal{O}(1)$. However, similar to the way in which a gauge boson propagator receives quantum corrections through loops of particle-antiparticle pairs, the mass parameter -μ^2 for the Higgs doublet H also gets such radiative corrections from loops involving all the particles that couple to H. Thus, the physical Higgs mass parameter μ_{phys} is actually the sum of the -μ^2 that we input into $\mathcal{L}$, referred to as the tree-level parameter, and a piece arising from quantum corrections. In the SM, the largest such correction comes from the top Yukawa coupling. Formally, computing the correction involves integrating over the momenta of the particles in the loop. Since there are no symmetries protecting the mass term of a scalar field, this integral diverges quadratically. This divergence signals that new physics must come in at some higher energy scale, and we parametrize this scale by introducing a cutoff for the integral. At best, the SM is valid up to M_{Pl}; choosing this as our cutoff yields $\mu_{phys}^2 = -\mu^2 + aM_{Pl}^2$, where a is some numerical coefficient that comes from computing the loop diagram, containing the Yukawa coupling and a factor of $1/16\pi^2$, known as a loop factor because it appears in all one loop calculations. Thus, we see that we need a large cancellation between the tree-level parameter and loop correction to now get μ_{phys} of order the weak scale. The incredible amount of fine tuning required is referred to as the hierarchy problem.

Clearly, explaining dark matter requires enlarging the field content. It seems reasonable that the other issues could possibly be addressed as well in this way. Supersymmetry is one such example, and is introduced in the next section.

1.2 Beyond the Standard Model: Supersymmetry

1.2.1 Introduction

Supersymmetry (SUSY) is a symmetry that relates fields of different spin via supersymmetry transformations. More precisely, the generators of such transformations are in the spin-1/2 representation of the Lorentz group, so SUSY relates fields with spins that differ by 1/2. Any extension of the SM that incorporates SUSY must at least double the amount of particles, since there is no way to relate the SM fields amongst themselves. Thus, a SUSY theory proposes partners for each of the known SM particles. These superpartners have the same quantum numbers and masses as the corresponding SM particles: SM fermions (spin-1/2) are paired with spin-0 (scalar) bosons referred to as sfermions; SM gauge bosons (spin-1) are paired with spin-1/2 fermions called gauginos, and the Higgs get spin-1/2 Higgsino partners.

All the fields that are mixed together by supersymmetry transformations can be grouped into a supermultiplet, which is similar to the way that one builds $SU(2)$ or $SU(3)$ gauge multiplets out of their up and down or color components. In this language, all matter fields can be described by chiral supermultiplets $\Phi = (\phi, \psi, F)$, where ϕ is a complex scalar (2 real degrees of freedom) and represents the sfermion, ψ is a left-handed two-component Weyl spinor, describing the spin-1/2 fermion, and F is an auxiliary complex scalar field that is needed to close the algebra and that can be eliminated using the equations of motion. The most compact formulation of supersymmetry, where supersymmetric invariance of all terms is manifest, is in superspace. Superspace is the manifold formed by adding to the usual four bosonic spacetime coordinates x^μ an additional pair of fermionic coordinates, i.e. Grassmann variables $(\theta, \bar{\theta})$, which are constant (complex) Weyl spinors. In this language, a superfield is a function of superspace, with infinitesimal translations in superspace corresponding to the supersymmetry transformations described above. A superfield can then be defined as a Taylor expansion in the fermionic coordinates, which terminates because the variables are Grassmann-valued. The expansion for a chiral superfield is

$$\Phi(\theta, \bar{\theta}) = \phi + \sqrt{2}\,\theta\psi + \theta^2 F. \tag{1.9}$$

The lack of $\bar{\theta}$ dependence follows from the chirality condition that defines the mutliplet. Although there is a beautiful geometric interpretation to superspace, we will not pursue that here, and content ourselves with using the superspace language as a tool for figuring out the Lagrangian interactions of the component fields, the exact procedure to be specified below.

Gauge fields are grouped into a vector supermultiplet V, which is defined by the reality condition $V = V^\dagger$. It has only three nonzero components (A_μ, λ, D) in a particular gauge (Wess-Zumino gauge), where A_μ represents the gauge boson, λ is a two-component Weyl spinor describing the gaugino, and D is an auxiliary real field that can be eliminated using the equations of motion. Formally, the Taylor expansion of a $U(1)$ gauge supermultiplet V is

$$V = \bar{\theta}\bar{\sigma}^\mu\theta A_\mu + \bar{\theta}\bar{\theta}\theta\lambda + \theta\theta\bar{\theta}\lambda^\dagger + \theta^2\bar{\theta}^2 D, \tag{1.10}$$

with the non-Abelian generalization obtained by putting the appropriate adjoint indices on the gaugino and gauge fields.

Having described the basic building blocks, we now explain the general procedure for constructing a supersymmetric Lagrangian. A key observation is that the mass dimension of θ and $\bar{\theta}$ is -1/2, so that fields found in higher order terms in the Taylor expansion have greater mass dimension. Thus, the F component of a chiral superfield has the highest mass dimension and can only transform as a total derivative (which vanishes on the boundary) and hence is invariant. Additionally, it can be shown that any holomorphic function of chiral superfields is itself a chiral superfield. The most general such function is denoted as $W(\Phi_i)$ and is called the superpotential. We obtain a SUSY invariant by projecting out its F component via an integral over the fermionic coordinates, $\int d^2\theta \, W(\Phi_i)$. Similarly, the D component of a vector superfield is invariant; we can form a vector superfield out of $\{\Phi_i, \Phi_i^\dagger\}$; the most general such function is denoted $\mathcal{K}\left(\Phi_i, \Phi_i^\dagger\right)$ and is called the Kähler potential. The SUSY invariant is obtained by taking its D component, i.e. $\int d^4\theta \, \mathcal{K}$, where $d^4\theta \equiv d^2\theta \, d^2\bar{\theta}$.

In general, one obtains from the Kähler piece the usual kinetic terms (familiar from ordinary field theory) for the fields. If there are local gauge symmetries, gauge invariance demands that we modify the Kähler potential in an appropriate way by introducing a function of the corresponding vector superfield, which in component language just promotes derivatives to covariant derivatives, as expected. The one new quantity in a supersymmetric theory arising from the Kähler potential is a D-term scalar potential $V_D(\phi^\dagger, \phi) = \frac{1}{2} D^a D^a = \frac{1}{2} g_a^2 (\phi^\dagger T^a \phi)^2$, where the last equation is obtained by solving for D^a using the equations of motion; the implied sum over a runs over all gauge groups, and for each group (which in general is non-Abelian), over all of the generators.

There is also a similar contribution from the superpotential. Defining a scalar "superpotential" $\mathcal{W}(\phi_i) = W(\Phi_i \to \phi_i)$, i.e. replacing all superfields by their scalar components, the F-term scalar potential is given by $V_F = F_i^\dagger F_i = |\partial \mathcal{W} / \partial \phi_i|^2$. Mass terms for component fermions and their Yukawa interactions are obtained from the superpotential as $-\frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial\phi_i\partial\phi_j}\psi_i\psi_j$. In summary then, for a supersymmetric theory

$$\mathcal{L} = \int d^4\theta\, \mathcal{K}(\Phi_i, \Phi_i^\dagger) + \int d^2\theta\, W(\Phi_i) \tag{1.11}$$

$$\mathcal{L} = \{\text{kinetic (derivative) terms}\} - \frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + V_F + V_D. \tag{1.12}$$

1.2.2 The Minimal Supersymmetric Standard Model (MSSM)

The Minimal Supersymmetric Standard Model (MSSM) is the simplest supersymmetric extension of the SM, in which all of the SM fermions are placed in chiral multiplets. Since in SUSY we work with left-handed Weyl spinors only but need to obtain the right-handed components of SM fermions as well, we must define oppositely charged left-handed fields, denoted with a bar, that we then charge conjugate, for example, $u_R = \bar{u}^c$. The MSSM field content is presented in Tables 1.4 and 1.5.

The MSSM superpotential is

$$W_{MSSM} = y^u_{ij}\bar{u}_i q_j H_u - y^d_{ij}\bar{d}_i q_j H_d - y^e_{ij}\bar{e}_i l_j H_d + \mu H_u H_d. \tag{1.13}$$

Note that in the MSSM we need two independent Higgs doublets, H_u and H_d, to get both up and down type Yukawa interactions, since W must be holomorphic. The doublets must then both get VEVs: $\langle H_u\rangle = v_u$, $\langle H_d\rangle = v_d$, with $v = \sqrt{v_u^2 + v_d^2}$, and it is useful to define the parameter $\tan\beta = v_u/v_d$ [2]. The last term above is the

[2] Actually, this all applies only to the weak-scale MSSM, where both doublets are light. See Sec. 1.4.2 for a different setup.

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	q	$(\tilde{u}_L\ \ \tilde{d}_L)$	$(u_L\ \ d_L)$	$(\ \mathbf{3},\ \mathbf{2}\ ,\ \frac{1}{6})$
(×3 families)	$\bar{u}$	$\tilde{\bar{u}}$	$\bar{u}$	$(\ \overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\bar{d}$	$\tilde{\bar{d}}$	$\bar{d}$	$(\ \overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	l	$(\tilde{\nu}\ \ \tilde{e}_L)$	$(\nu\ \ e_L)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ -\frac{1}{2})$
(×3 families)	$\bar{e}$	$\tilde{\bar{e}}$	$\bar{e}$	$(\ \mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(\tilde{H}_u^+\ \ \tilde{H}_u^0)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ +\frac{1}{2})$
	H_d	$(H_d^0\ \ H_d^-)$	$(\tilde{H}_d^0\ \ \tilde{H}_d^-)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ -\frac{1}{2})$

Table 1.4: The matter and Higgs sectors in the MSSM, represented by chiral supermultiplets (adapted from [2]). The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions, though the L subscript is only used above to indicate the fields that transform non-trivially under $SU(2)_L$.

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\tilde{g}$	g	$(\ \mathbf{8},\ \mathbf{1}\ ,\ 0)$
winos, W bosons	$\tilde{W}^\pm\ \ \tilde{W}^0$	$W^\pm\ \ W^0$	$(\ \mathbf{1},\ \mathbf{3}\ ,\ 0)$
bino, B boson	$\tilde{B}^0$	B^0	$(\ \mathbf{1},\ \mathbf{1}\ ,\ 0)$

Table 1.5: Gauge supermultiplets in the MSSM (adapted from [2]).

supersymmetric version of the SM Higgs boson mass term, and in the limit of small mixing with the electroweak gauginos, is the Higgsino mass. Using the prescription in Eq. (1.12) to obtain the component terms, we see that we obtain the SM-like fermion-fermion-Higgs interactions similar to those in Eq. (1.4), as well as the supersymmetrized versions of these, fermion-sfermion-Higgsino couplings. Additionally, there are sfermion-sfermion-Higgs and sfermion-sfermion-Higgs-Higgs interactions in V_F. Since each particular set of such interactions is derived from the same term in the superpotential, all of these depend on the same Yukawa coupling.

This simple observation has profound consequences for the Higgs mass in a supersymmetric theory, because it turns out that if SUSY is unbroken, the quantum corrections to μ vanish! This is because, in addition to the top loop corrections described above, there are now also loops involving stops, which are proportional to the same top Yukawa coupling. The stop loops have opposite sign because the stops are scalars, and so have opposite statistics to the fermionic tops. Thus, supersymmetry provides an elegant solution to the hierarchy problem, and for a long time this has guided most SUSY model builders. It should be noted that the Higgs mass is an example of a more general feature of the superpotential in unbroken supersymmetric theories: none of the parameters get quantum corrections. This is known as the "non-renormalization theorem" [10, 11].

However, since sfermions (or any other SUSY particles for that matter) have yet to be discovered, they must have masses that are larger than those of the SM fermions, thus breaking the symmetry. To preserve the cancellation of quadratic divergences, the relationships between dimensionless couplings that exist in an unbroken supersymmetric theory must still hold, e.g. stop loops must still depend on the same top Yukawa coupling. This leads to the notion of "soft" supersymmetry breaking: The effective MSSM Lagrangian is written in the form

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}, \tag{1.14}$$

where $\mathcal{L}_{SUSY}$ contains all the interactions that preserve SUSY, while $\mathcal{L}_{soft}$ parametrizes the SUSY breaking, containing additional mass terms for the unobserved superpartners, i.e. all sfermions, gauginos, and Higgsinos, as well as all other terms of positive mass dimension allowed by the gauge symmetries, including the SUSY-breaking analogue of the last term in Eq. (1.13), $B\mu\, H_u H_d$ (here the fields are just the scalar components of the supermultiplets). The requirement of positive mass dimension ensures that the non-supersymmetric corrections to the Higgs mass diverge at most logarithmically, with this following from simple dimensional analysis, as the corrections must be proportional to the soft parameters themselves, since they vanish in the limit that the soft parameters go to zero and SUSY is recovered.

Turning our attention to the Higgs sector in the MSSM, we see that the Higgs potential will depend on $|\mu|^2$, $B\mu$, and soft masses for H_u and H_d, $m_{H_u}^2$, $m_{H_d}^2$. Minimizing this potential should yield a weak scale VEV, so to have a natural solution to the hierarchy problem and avoid tuning, all of these parameters should also be of the same order. This is the motivation behind the paradigm of the *Natural (weak-scale)*

MSSM, where in fact all of the parameters in $\mathcal{L}_{soft}$ are required to lie close to the weak scale, for the following reasons. Although the sfermion and gaugino masses do not factor into the minimization conditions, sfermions and electroweak gauginos appear in the one-loop corrections to the Higgs soft masses, which are proportional to either the sfermion or gaugino mass, and squark masses themselves are corrected at two-loop order by the gluino mass.

However, μ is a conceptually different parameter, since it respects SUSY, so there is no reason a priori for it to be of the same order as the SUSY-breaking parameters, but it must be to avoid tuning. The most common solution to this puzzle, then, is to assume that the μ term is absent at tree-level and arises only after SUSY breaking, so that its size is naturally of the same order as that of the soft parameters. Note, however, that μ must be generated in some way because the Higgsinos must have masses at least $\gtrsim 100$ GeV to satisfy the bound from the Large Electron Positron Collider (LEP) on the lightest chargino (the lightest particle among the charged winos and Higgsinos).

Since there are two Higgs doublets in the MSSM, there are initially eight real degrees of freedom. However, after EWSB, three of these become the longitudinal polarizations of gauge bosons, as described in Sec. 1.1, leaving five physical degrees of freedom: a charged Higgs $H^{\pm}$, two CP-even scalars H^0 and h^0 (CP is the product of charge conjugation and parity, both discrete symmetries associated with spacetime), and a CP-odd scalar A^0. Since the lightest of these is h^0, this is the one most analogous to the SM Higgs h. In fact, while the masses of the others can be taken arbitrarily large by increasing m_{A^0}, the tree-level mass of h^0 is bounded from above by essentially the mass of the Z boson, $m_Z \approx 90$ GeV. Additionally, in the "decoupling" limit $m_{A^0} \gg m_Z$ in which the bound is saturated, the couplings of h^0 to fermions and gauge bosons are identical to those of the Higgs h without supersymmetry. As explained above, this tree-level mass is subject to quantum corrections, which depend on the degree of SUSY breaking in each multiplet that couples to the Higgs. More precisely, the correction scales as the logarithm of the ratio of the sfermion mass to the fermion mass. As in the SM, the dominant contribution comes from diagrams that involve the top Yukawa coupling.

SUSY theories also have additional appealing properties: 1) a natural dark matter candidate and 2) improved gauge coupling unification. Models of SUSY based on R-parity, a discrete symmetry, have a lightest superpartner (LSP) that is stable and can have the properties of WIMP dark matter, with the lightest neutralino a particularly appealing candidate. In the weak-scale MSSM, this is either the neutral wino or Higgsino. To get the observed dark matter abundance, the annihilation cross section has to assume a certain value, and this value depends on the WIMP mass. For a neutralino WIMP that accounts for all of the dark matter, this is $\sim$ a few TeV, which is more-or-less consistent with a natural theory. R-parity is motivated by the need to forbid certain terms in the superpotential, allowed by all the gauge symmetries, that would lead to proton decay at a rate larger than the experimental bound.

Under this symmetry, all the unobserved superpartners are odd, so they can only be produced in pairs–hence the lightest one cannot decay. A decay to a neutralino LSP is characterized experimentally by a significant amount of missing transverse energy (MET). Namely, the LSP, interacting only weakly, passes right through the detector and so there is a large amount of energy that is not accounted for when the decay event is reconstructed. In addition, the extra particle content of SUSY models leads to better unification of the gauge couplings of the three SM forces at around the grand unification scale $M_{GUT} \sim 10^{16}$ GeV.

1.3 Supersymmetry Breaking

Although SUSY breaking is introduced explicitly in the MSSM, SUSY must be broken spontaneously so that the Lagrangian takes the form of Eq. (1.14), with the terms in $\mathcal{L}_{soft}$ satisfying the requirements mentioned in the previous section. This means that although the equations of the theory are supersymmetric, the vacuum about which we define excitations is not invariant under such transformations. The scalar potential $V = V_F + V_D$ in a SUSY theory is positive-definite, and it can be shown that all SUSY preserving vacua have zero energy. Thus, SUSY is spontaneously broken if there exist multiplets with non-vanishing F or D vacuum expectation values (VEVs). It turns out, however, that it is not acceptable for any the MSSM multiplets to develop such VEVs. Explaining the source of SUSY breaking thus requires extending the MSSM and then describing how the breaking is communicated.

It appears that the simplest possibility would be to couple the MSSM multiplets directly to the source of SUSY breaking by writing down tree-level (i.e. renormalizable, operator coefficients have positive mass dimension) interactions in the superpotential or Kähler potential. However, this is problematic for two reasons. First, due to the structure of SUSY theories, gauginos would not get masses in this way. Second, in this case there are tree-level sum rules that simply relate the sfermion masses to those of the fermions, implying light squarks and sleptons that have not been observed. For these reasons, the terms in $\mathcal{L}_{soft}$ must arise indirectly or through quantum corrections. The standard SUSY breaking paradigm therefore consists of a "hidden" sector that is not directly coupled to the "visible" sector of the MSSM and that contains the multiplet(s) developing F or D term VEVs. However, the two sectors must share some interactions that mediate the SUSY breaking from the hidden sector to the visible one.

1.3.1 Gravity Mediation

The simplest candidate for a mediator is gravity, since it couples to everything (all matter and energy). Thus, if the hidden sector field(s) do not have SM interactions, this is the dominant source of SUSY breaking. Since gravitational interactions only become important at the scale M_{Pl}, the idea here is that SUSY breaking is derived from the new physics that enters at this scale. Parametrizing the SUSY breaking by

F_X, the F term VEV of the hidden sector field X, scalar masses arise from the higher dimensional Kähler potential operators of the form

$$\lambda_{ij} \int d^4\theta \, \frac{X^\dagger X \Phi_i^\dagger \Phi_j}{M_{Pl}^2}, \tag{1.15}$$

implying scalar masses

$$m_{sc} \sim F_X / M_{Pl}. \tag{1.16}$$

However, in general the coupling matrix λ_{ij} is not diagonal and so will generate $\mathcal{O}(1)$ mixings in the mass matrix between different flavors, allowing for flavor-changing neutral current (FCNC) effects. In general, FCNC processes are transitions that involve a change in the flavor of the quark(s) or lepton(s) in the initial state while preserving the charge, and are present to some degree in all SUSY models. Having relatively small SM contributions, they have been tightly constrained. Examples include lepton flavor violation, e.g. $\mu \to e\gamma$, neutral meson mixing, such as in the kaon sector, and contributions to electric dipole moments (EDMs) of quarks or leptons. Since in all of these processes the sfermions appear in loops, the size of the contribution scales inversely with the sfermion mass. Thus, "natural" scalar masses, i.e. close to the weak scale, arising from gravity mediation have long been ruled out by experiment due to the "anarchic" flavor mixing.

One can also write down a higher dimensional operator in the superpotential that gives masses to the gauginos:

$$\int d^2\theta \, \frac{X W^\alpha W_\alpha}{M_{Pl}}, \tag{1.17}$$

where W_α is the SUSY analogue of the gauge invariant field strength $F_{\mu\nu}$ and is constructed out of V. However, this operator is only invariant under all symmetries if X is a singlet under all gauge and global symmetries. Many models of SUSY breaking do not feature such singlets, in which case, assuming that gravity is the only mediator, gauginos get masses from the mechanism of anomaly-mediated supersymmetry breaking (AMSB).

1.3.2 Anomaly Mediation (AMSB)

The soft masses above were derived simply by "integrating out" new physics at the scale M_{Pl}, obtaining an effective theory below this scale. However, it is possible that the SUSY breaking itself is communicated by the VEV of an auxiliary field in the gravity multiplet. Supergravity (SUGRA) is the supersymmetric generalization of general relativity, with local supersymmetry transformations, where the superpartner of the spin-2 graviton is the spin-3/2 gravitino. Once SUSY is spontaneously broken, the gravitino acquires a mass

$$m_{3/2} \sim F_X/M_{Pl}. \tag{1.18}$$

Although there are several different (equivalent) formulations of SUGRA, perhaps the simplest is the superconformal tensor calculus approach. To motivate this, we can recall that general relativity (GR) can be rewritten as a theory with an extra symmetry, invariance under local scale transformations (spacetime dependent rescalings of the metric) by introducing an extra scalar field. This scale invariance then implies invariance under a larger symmetry group, the set of conformal symmetries. Thus, in the superconformal approach, one writes down a supersymmetric theory of gravity invariant under conformal transformations. This theory includes the standard gravity multiplet, containing the graviton and gravitino, among other fields, but having no scalars. Additionally, the scalar field from conformal GR is promoted to a chiral multiplet, the conformal compensator multiplet ϕ. Thus, the SUSY breaking can only be transmitted by the F term VEV F_ϕ of the compensator multiplet. The advantage of the superconformal approach then is that the coupling of matter to SUSY breaking, i.e. to the multiplet ϕ, is completely dictated by demanding superconformal invariance.

The spontaneous breaking of SUSY by the hidden sector VEV F_X also breaks superconformal invariance, so the conformal compensator acquires the F term VEV

$$F_\phi = m_{3/2}. \tag{1.19}$$

Assuming for now that the μ term is absent from the MSSM superpotential, there are no dimensional parameters in the MSSM, so conformal invariance is preserved at the classical (tree) level. However, quantum (loop-level) effects require introducing a renormalization scale (or equivalently a regulator), explicitly breaking the symmetry. Hence, the AMSB soft masses arise at loop-level: gaugino masses are generated at the one-loop level and the scalar masses squared are generated at two loops. The equations for these soft masses are actually exact to all orders in perturbation theory– they are the solutions to the RGEs for the gauginos and scalars and thus are valid at all energy scales, defining an AMSB trajectory. The gaugino masses are given by

$$m_i = \frac{\beta(g_i)}{g_i} m_{3/2} \tag{1.20}$$

Since this formula is valid at all scales, if we wish to calculate the masses at some scale we only need to know the values of the couplings at that scale. Using the one-loop beta functions, we obtain

$$m_1 = 11 \frac{\alpha_1}{4\pi} m_{3/2} \tag{1.21}$$

$$m_2 = \frac{\alpha_2}{4\pi} m_{3/2} \tag{1.22}$$

$$m_3 = -3 \frac{\alpha_3}{4\pi} m_{3/2}. \tag{1.23}$$

Note that here we are using the non-GUT normalization for hypercharge. Plugging in the weak scale values of the α_i, we find that the mass ratios $m_1 : m_2 : |m_3|$ are approximately $3.3 : 1 : 10$. We assume here that μ is larger than any of the gaugino masses, so that the wino is the LSP in this minimal case. This large splitting between the gluino and the wino means that the quarks produced in the cascade decay of the gluino will be very energetic. However, these quarks are not directly observed but rather hadronize, as explained in Sec. 1.1, each producing a "jet" of color-neutral particles. Such energetic jets are easy to detect experimentally, so the dearth of such events leads to tight constraints on the gaugino masses.

The scalar masses in AMSB are given by

$$m_i^2 = -\frac{1}{4} \left(\frac{d\gamma_i}{dg_j} \beta_{g_j} + \frac{d\gamma_i}{dy_j} \beta_{y_j} \right) m_{3/2}^2, \tag{1.24}$$

where γ is the corresponding anomalous dimension (related to wavefunction renormalization), and β_g, β_y are the gauge coupling and Yukawa coupling beta functions, respectively. These masses are flavor diagonal, i.e. there is no mixing, so AMSB is flavor blind, as opposed to the gravity mediation discussed in the previous section. Nonetheless, we see that gravity mediation is the dominant contribution to the scalar masses unless it is suppressed in some way. In fact, the flavor problems associated with gravity mediation motivated a lot of AMSB model-building with such a suppression mechanism, known as sequestering. Finally, since the sleptons are charged only under non-asymptotically free gauge groups (which have positive beta functions), they will have negative squared masses. This is unacceptable because such masses in general would induce the breaking of $U(1)_{EM}$, and therefore this is referred to as the tachyonic slepton problem in AMSB.

We now return to the μ term in the MSSM superpotential. If present at tree-level, it explicitly breaks the superconformal invariance, and so generates at tree-level the corresponding soft term $B\mu \sim \mu m_{3/2}$. To achieve proper EWSB, the Higgs sector parameters must roughly obey $B\mu \sim \mu^2$. However, in the natural MSSM, the Higgsinos should be at the same scale as the gauginos, so $\mu \sim m_{3/2}/16\pi^2$. Thus, we see that even if one puts in a μ term by hand (without explaining its value) in minimal AMSB, the resulting $B\mu$ is a loop factor too large for proper EWSB. This difficulty is known as the μ problem in AMSB.

There is an elegant solution to the μ problem in theories with gravity mediation, known as the Giudice-Masiero (GM) mechanism [12]. The simplest way to understand it is to consider the Kähler potential operator

$$\lambda \int d^4\theta H_u H_d \tag{1.25}$$

which vanishes in ordinary (global) low-energy supersymmetry. However, in the superconformal formulation of SUGRA (local supersymmetry), all chiral superfields, which normally have a mass dimension of 1, are treated as dimensionless, with the conformal compensator multiplet ϕ "carrying" the dimensionality. Thus we must modify Eq. (1.25) to

$$\lambda \int d^4\theta\, \phi^\dagger \phi H_u H_d \longrightarrow \lambda \int d^4\theta\, \frac{\phi^\dagger}{\phi} H_u H_d, \tag{1.26}$$

where in the second step we recover the usual dimensionality by absorbing ϕ: $H_{u,d} \to \phi\, H_{u,d}$. Plugging in for the VEV of the compensator, $\phi = 1 + \theta^2 m_{3/2}$, into Eq. (1.26), we see that this generates $\mu = \lambda m_{3/2}$ and $B\mu = \lambda m_{3/2}^2$. With $\lambda \sim \mathcal{O}(1)$, $B\mu \sim \mu^2$, so we can get viable EWSB. However, in the case that gauginos get masses from AMSB, we need $m_{3/2} \sim 100$ TeV, so this mechanism is not satisfactory in the context of the natural MSSM, since one would then have to tune to get the right Higgs mass.

1.3.3 Gauge Mediation (GMSB)

Given the problems with gravity mediation, one might consider the SM gauge interactions instead; this is known as gauge-mediated supersymmetry breaking (GMSB).

In this setup, one introduces heavy chiral multiplets Ξ, $\overline{\Xi}$ that are oppositely charged under the SM gauge groups and hence are referred to as a vector-like messenger pair. To preserve gauge coupling unification, these messengers must be introduced in complete representations of $SU(5)$. They are directly coupled in the superpotential to the source of SUSY breaking through the term

$$\int d^2\theta X \, \Xi \, \overline{\Xi}, \tag{1.27}$$

$$X = M + \theta^2 F_X, \tag{1.28}$$

so the messengers have a supersymmetric mass M and there is a SUSY breaking mass mixing F_X between the scalars from the two multiplets. Since the messengers are charged under the SM gauge groups, they couple to gauginos, so one can draw a one loop diagram that corrects each gaugino propagator. As they are heavy, they can be integrated out, and this yields non-zero threshold corrections to the gaugino masses because both messenger scalars and fermions appear in the loop, and have different masses. For one messenger pair, the gaugino masses are given by

$$m_i = \frac{\alpha_i}{4\pi} \frac{F_X}{M}. \tag{1.29}$$

The scalar masses squared arise at two-loop order:

$$m_{sc}^2 = \sum_i \left(\frac{\alpha_i}{4\pi} \frac{F_X}{M} \right)^2, \tag{1.30}$$

where the sum is over all the SM gauge groups to which the scalar couples. Of course, we also have the gravity-mediated contributions of Eq. (1.15); however, these are negligible because they are suppressed by $M_{Pl} >> M$.

Since the SM gauge interactions are flavor blind, GMSB, like AMSB, does not introduce any flavor mixing. However, GMSB has a μ problem as well. Assuming that the μ term is absent at tree-level, one must introduce additional interactions between the Higgs multiplets and the messengers to generate μ as well as $B\mu$. This is because without the μ term, the MSSM superpotential has an additional symmetry that is not broken by gauge interactions. However, in this case $B\mu$ arises radiatively at the same loop order as μ, meaning that $B\mu/\mu^2 \sim 16\pi^2$, too large for proper EWSB.

1.4 Tension between Experiment and Naturalness

Data from the LEP collider constrained the Higgs mass to lie above about 115 GeV, which in the weak-scale MSSM can only be achieved with stops close to 1 TeV, about a factor of 10 larger than m_Z. This implies that some moderate amount of tuning is already required in order to obtain the right value of v, and thus is in tension with the idea of SUSY as a natural solution to the hierarchy problem that inspires the weak-scale MSSM. As we have seen in the discussion on SUSY breaking, in general the other superpartners will have masses roughly close to the stops. However, since the LEP shutdown in 2000, newer generations of colliders have been quickly raising the bounds on superpartner masses based on the non-observation of the large missing energy events predicted in the R-parity conserving MSSM. For example, the null results from such searches based on analyses of the 7 TeV data [13, 14] collected at the LHC and the 8 TeV data [15–18] have placed a lower bound on the gluino mass of about 1.5 TeV for many vanilla models.

1.4.1 Gaugomaly Mediation

This was the context in which the work described in Ch. 2 began, motivated by the following general considerations. Most generic models of SUSY breaking do not feature singlets, so gauginos get masses from AMSB. To maintain naturalness, sfermion masses should also be of the same order, meaning that the dominant contribution to their masses should also come from AMSB. However, as mentioned in the section above, AMSB suffers from the tachyonic slepton problem. In addition, the inherently large splitting between the gluino and LSP wino masses in minimal AMSB implies numerous events with lots of missing energy. To resolve these issues, we introduce messenger fields familiar from GMSB into the AMSB framework, obtaining a hybrid theory of gauge and anomaly mediation. We find that any number of vector-like messenger fields (allowed by GUT unification) compress the predicted gaugino spectrum when their masses come from the Giudice-Masiero mechanism. This more compressed spectrum is less constrained by LHC searches and allows for lighter gluinos.

1.4.2 Split SUSY

The recent announcement by the CMS and ATLAS collaborations of the discovery of a Higgs-like particle with a mass of about 126 GeV requires superpartners with masses in the range of $5 - 10$ TeV [19], raising serious doubts about naturalness. Thus, perhaps one should not insist on naturalness as a lead in constructing SUSY models, but instead focus on its other advantages, namely a dark matter candidate and gauge coupling unification. This is the approach taken in the framework of *Split Supersymmetry* [20–22], which abandons the idea of SUSY as a solution to the hierarchy problem. The original framework was developed several years before the Higgs announcement, so the notion of heavy scalars was motivated by the theoretical observation that there should generically exist a hierarchy between scalar and gaugino masses since it is harder to give gauginos masses–one first has to break a type of $U(1)$ symmetry implicit in SUSY known as R-symmetry (not to be confused with R-parity!).

The original models therefore had heavy scalars with common mass m_{sc} (masses assumed degenerate for simplicity) ranging from 100 TeV all the way up to scales approaching M_{GUT}, and light gauginos and Higgsinos of about 1 TeV, with this scale motivated by WIMP dark matter. In these models, the Higgsinos were kept light with an approximate R symmetry or another global $U(1)$ symmetry (Peccei-Quinn), although we will see that this is not the most natural scenario. The heaviness of the scalars requires one to assume that the hierarchy problem is solved with fine tuning, i.e. one linear combination of the Higgs doublets, $H_1 = -\cos\beta(i\sigma_2)H_d^* + \sin\beta H_u$, is tuned to be light while the orthogonal combination is heavy [3]. This is just the extreme version of the "decoupling" limit for the Higgs mass eigenstates mentioned in Sec. 1.2.2, so the properties of H_1 are identical to those of the SM Higgs doublet H. It is important to point out here that the need to tune is not really a new feature of SUSY models. It is already present in most weak-scale models of SUSY, where the only viable explanation for the smallness of the cosmological constant (related to dark energy) appears to be fine-tuning. Such a tuning is motivated by the anthropic principle [23], a vacuum selection mechanism acting on the enormous landscape of vacua predicted by string theory [24, 25]. Furthermore, anthropic arguments can be made for the value of the weak scale [26], making the idea of fine-tuning close to the TeV scale more plausible.

The upshot of such heavy scalars is that they strongly suppress flavor-changing neutral currents, which are a major problem in the natural MSSM, requiring somewhat contrived solutions in that context. The light gauginos/Higgsinos serve as good dark matter candidates while the heavy scalars do not hurt gauge coupling unification because the squarks and sleptons now missing in the running from the weak scale to m_{sc} come in complete $SU(5)$ multiplets.

In terms of experimental signatures, the smoking gun of Split SUSY is a displaced gluino vertex. Gluinos produced at colliders can only decay to the LSP through heavy

[3] In Split SUSY, $\tan\beta$ is simply a rotation factor describing the mixing, and is not a ratio of Higgs VEVS as in the weak-scale MSSM, since there are no VEVs at the high scale where the heavy doublet is removed.

off-shell (virtual) squarks, so it is possible that they can travel macroscopic distances of about 1 mm or more before decaying. For this vertex to be visible using current techniques, the squarks must have masses

$$m_{\tilde{q}} \gtrsim \left(\frac{\Delta m}{1\ \text{TeV}}\right)^{5/4} \times (500\ \text{TeV}), \tag{1.31}$$

where Δm is the mass difference between the gluino and the LSP. So with $m_{\tilde{q}} \sim m_{sc} \sim \mathcal{O}(100\ \text{TeV})$, a visible displaced vertex requires $\Delta m \lesssim 100$ GeV.

1.4.3 Mini-Split SUSY

Although the original models of Split SUSY may seem arbitrary since m_{sc} is no longer constrained by naturalness, a concrete picture emerges when one tries to accomodate into this framework a 126 GeV Higgs mass. This mass points to the most moderate scalar masses in Split SUSY, with $m_{sc} \sim 100 - 1000$ TeV. Since we need gaugino masses to be in the range of 1 - 10 TeV for a WIMP dark matter candidate, we see that we can realize Split SUSY in perhaps the simplest possible way, in the context of gravity mediation with a SUSY breaking field that is not a singlet. In this case, $m_{sc} \sim m_{3/2} \sim 100-1000$ TeV, so the gaugino masses from AMSB automatically come out in the desired range. In fact, gaugino masses that are just a loop factor down from the scalars are perhaps the most natural, since additional suppression would need to be explained with some extra theoretical machinery besides the breaking of a symmetry. Thus, aside from μ, which we have not discussed, all the soft masses are set by one parameter, $m_{3/2}$.

Sticking with this minimalist philosophy, we might wonder if μ should not also be set by the same scale, since there is no compelling symmetry, such as an exact R symmetry, protecting it. In fact, we know from Sec. 1.3.2 that we can achieve this simply through the Giudice-Masiero mechanism, which is used in Ch. 2 to give masses to the messengers. Thus, the Higgsinos are heavy, and this is a departure from the original Split SUSY construction. We refer to it as *Mini-Split SUSY* and explore its properties in Ch. 3, where we show that in this framework one can obtain the right Higgs mass and good gauge coupling unification.

Since the most promising way to confirm Split SUSY in the near future would be to observe a displaced gluino vertex, it is important to understand the gaugino spectrum. With gaugino masses in the $1-10$ TeV range and the hierarchy of minimal AMSB presented earlier, we see that the splitting Δm is too large for a displaced vertex to be visible. Therefore, in Ch. 3 we also examine an extension of Split SUSY that incorporates messengers to compress the spectrum in a way analogous to what is done in Ch. 2.

Although there is large flavor mixing, all experimental bounds on FCNCs can be trivially satisfied because the sfermions are so heavy. In fact, this large flavor mixing can be recast as an advantage because it can be exploited to construct a radiative model of flavor. As described below, such models require additional field content beyond that of the MSSM, so this is additional motivation for the messengers that compress the gaugino spectrum. Although it might seem that such a solution to the flavor puzzle is generic to all models with heavy sfermions, the flavor model that we propose in Ch. 4 relies crucially on the fact that the splitting between the gaugino and sfermion masses is just a loop factor and not more.

1.4.4 Radiative Models of Flavor and Mini-Split SUSY

Unlike the gauge sector, the SM flavor sector has a complicated menagerie of dimensionless parameters whose values differ by orders of magnitude. However, the patterns of masses and mixings of the SM fermions do not appear random, even on a logarithmic scale; there is a hint of structure that emerges upon close inspection (*cf.* Fig. 1.1). For example, the masses of the 3^{rd} generation fermions are all much larger than the masses of the 2^{nd} generation fields with the same quantum numbers, which in turn are all much heavier than the corresponding 1^{st} generation fermions. As already discussed, one of the shortcomings of the SM is that it offers no explanation for any of this structure, with the Yukawa couplings simply given as dimensionless inputs.

One possible explanation for the flavor structure stems from the following observation about, for example, the up, charm, and top quarks

$$\frac{m_c}{m_t} \simeq \frac{m_u}{m_c} \simeq \mathcal{O}(1) \times \frac{1}{16\pi^2} \ . \tag{1.32}$$

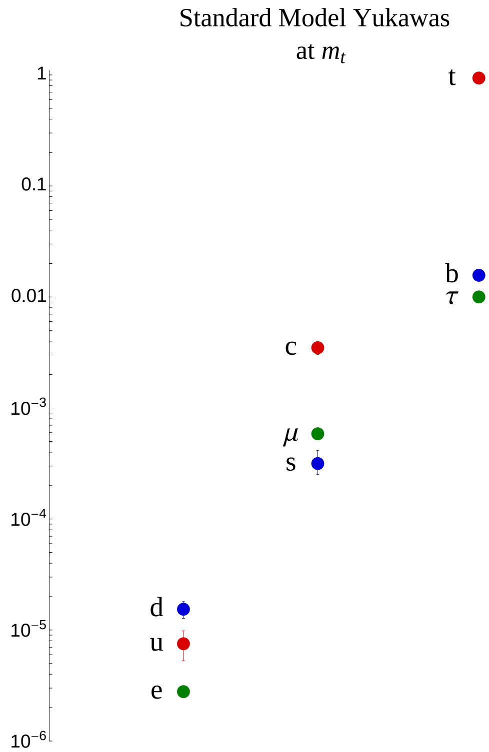


Figure 1.1: The hierarchy of SM Yukawa couplings on a logarithmic scale. We take the running-mass values at the top pole-mass (173.2 GeV) reported in [1] and divide by $v = 174.1$ GeV, as used in [27].

This leads to the idea of radiative flavor breaking [4] where only the 3rd generation Yukawa couplings are generated at tree level, with couplings for the lighter generations coming from quantum corrections: The 2nd generation Yukawas are generated as one-loop effects, and the 1st generation is a two-loop effect. This is an old idea that has its origins in trying to explain the electron mass as a loop effect of the muon mass [28–31] and that has since inspired a lot of non-supersymmetric model-building [32–45].

[4]The breaking referred to here is the explicit breaking of the $U(3)^5$ global flavor symmetry of the SM Lagrangian without the Yukawa terms of Eq. (1.4), i.e. invariance under transformations that rotate between the three generations of each of q, u, d, l, e.

On the other hand, supersymmetric theories of radiative flavor generation [46–53] automatically come with the additional benefits of SUSY, including a natural dark matter candidate and improved gauge coupling unification. However, SUSY also has features that specifically facilitate radiative flavor generation. The non-renormalization theorems for the superpotential guarantee that radiative corrections cannot generate new operators such as 1^{st} and 2^{nd} generation Yukawa couplings. This forces flavor and SUSY breaking to be tied together, likely giving a common scale to both phenomena. In addition, SUSY requires the theory to include an additional set of particles which transform under flavor, the sfermions. While non-supersymmetric theories of radiative flavor generation require introducing a host of new fields, SUSY models are potentially more economical because the sfermions can contribute to generating the flavor hierarchy.

In order to use the sfermions to generate flavor, there must be large flavor breaking in the sfermion sector. Unfortunately, if sfermions are at the weak scale, low energy flavor tests require them to be nearly flavor diagonal [2], a difficulty encountered by many of the early attempts to build such a model [46, 51]. Because the Yukawa couplings are dimensionless parameters, they are quite insensitive to the scale at which they are generated. On the other hand, the flavor observables that constrain the flavor breaking in the sfermion sector correspond to higher dimension operators, with coefficients scaling as $1/m_{sc}^n$, with $n \geq 1$, so they decouple quickly with heavier sfermion masses. Therefore, spectra where the sfermions are much above the weak scale, such as in Split SUSY, can be used for radiative flavor generation with sfermions potentially as heavy as the GUT or Planck scale [52].

The inspiration for our model comes from an observation made in Ch. 3 that 1$^{\text{st}}$ generation Yukawa couplings generated from corrections involving MSSM fields alone naturally come out roughly of the right size in the Mini-Split framework, for the following reason. The loop diagrams responsible for these corrections involve sfermions and gauginos and can be drawn at one-loop order. However, they require a gaugino mass insertion, so they are suppressed by a factor of $m_{\tilde{g}}/m_{sc}$. In Mini-Split, this is just a loop factor, so numerically these diagrams are of two-loop order, as they should be in the cascading paradigm discussed above. While the anarchic flavor structure of the sfermions can generate the 1$^{\text{st}}$ generation masses at loop level from those of the 3$^{\text{rd}}$ generation, more structure is necessary to generate the full SM spectrum. This additional structure depends on the nature of the additional symmetry that must be introduced to forbid the MSSM Yukawa couplings, since they are allowed by all the gauge symmetries.

The simplest such flavor symmetry is a $U(1)_F$, which is what we use in the model. Charges must be assigned to the Higgs and MSSM matter multiplets such that the Yukawa couplings are not allowed. We believe that a full explanation of the flavor hierarchy demands democratic treatment of all generations, so we leave all generations uncharged under this symmetry. It should be noted that most models have instead relied on so-called horizontal symmetries under which the various generations are treated differently. In our model, there are however messenger fields charged under $U(1)_F$ that have $\mathcal{O}(1)$ tree-level couplings to the Higgs doublets, i.e. primordial Yukawa couplings that, in effect, start the cascade. These messengers are coupled to the MSSM matter in the superpotential by way of flavon superfields, which also have non-zero charge.

Upon SUSY breaking, the scalars from these flavon multiplets get VEVs, breaking the flavor symmetry. This triggers mixing between one generation of quarks/leptons and the messenger fermions, leading to an $\mathcal{O}(1)$ Yukawa coupling for this generation, which we identify as the top/bottom/tau, depending on the sector. Proceeding onwards, a top Yukawa can, for example, seed an up coupling as described above, but obtaining the correct 2^{nd} generation (in this case, charm) Yukawa is more difficult. Although the role of flavons in most of the literature has been limited to breaking the flavor symmetry, we find that by introducing additional interactions between the flavon multiplets in the superpotential, we are also able to utilize the fermionic components of these multiplets as well as the dynamical scalars in a one loop diagram to generate 2^{nd} generation Yukawa couplings of appropriate size. With the Yukawa couplings in hand, we are also able to reproduce the correct structure for the CKM matrix.

Chapter 2

Gaugomaly Mediation Revisited

2.1 Introduction

In this chapter, we present a model that combines gauge and anomaly mediation to solve the tachyonic slepton problem and that also relaxes the constraints on gaugino masses by compressing the spectrum. This hybrid approach is not new. Refs. [54, 55] first showed that the D-type gauge mediation of Poppitz-Trivedi [56] could be simply combined with AMSB to solve the tachyonic slepton problem, although they did not specify the origin of the messenger masses. Ref. [57] studied this further and gave it the name 'gaugomaly' mediation. A less direct solution to the slepton problem was proposed in ref. [58], which developed extended anomaly mediation (EAM) by arranging for the messengers to get masses directly from anomaly mediation through Giudice-Masiero (GM) type terms [12]. This deflects the gaugino masses off of the

AMSB trajectory, changing the scalar masses through running. The EAM setup was itself extended in [59, 60] with the addition of a singlet to yield realistic spectra.

We take the approach that such singlets are unnatural, so we are led to consider EAM with the D-type GMSB of gaugomaly mediation, which surprisingly had not been explored previously. In addition, we investigate the effect of the messengers in compressing the gaugino spectrum, an interesting aspect not discussed in the references above that deserves attention on its own. Integrating out the messengers takes the gaugino masses off the AMSB trajectory and gives threshold corrections that modify the masses at leading order, with the ratios sensitive to the number of messenger pairs. The result is a compressed gaugino spectrum with the mass splitting between the gluino and the LSP depending on the number of messenger pairs. The limits on the allowed gaugino masses are significantly weakened due to the squeezing of the spectrum [61]. This framework can also bring models that are otherwise beyond the reach of the LHC to within its reach in certain cases as the gluino becomes only 1-2 times heavier than the LSP (the wino or the bino depending on the number of messengers). In addition to the model, we present gaugino pole mass equations that differ from (and correct) the original literature.

This chapter is structured as follows. First, we review the calculation of the gaugino pole masses in the minimal AMSB framework. Although this has already been discussed in [62], we present the details here because our equations differ slightly. We introduce messengers in Sec. 2.3 in the context of D-type gauge mediation as a solution to the tachyonic slepton problem. In Sec. 2.4, we discuss the gaugino spectra and related phenomenology independently from the scalars, focusing on the compression resulting from the messenger threshold corrections to the gaugino masses. Then, in Sec. 2.5, assuming that the μ problem has been solved, we give complete example spectra for our model with the scalars. We conclude in Sec. 2.6 by presenting a simple approach to addressing the μ problem in this framework and briefly discuss how it can be improved by incorporating some ideas found in the literature.

2.2 Gaugino Pole Masses

The full NLO expression includes contributions coming from α_3 and y_t in the two-loop beta functions as well as self-energy corrections. For the one-loop self-energies, our analysis follows closely the steps presented in [63].

2.2.1 Gauge Loops

The contribution to the self-energy from gauge boson loops is given by

$$\left(\frac{\Delta m_i}{m_i}\right)_{gauge} = \frac{\alpha_i}{4\pi} C(G_i) \left[4B_0(m_i, m_\psi, m_\phi) - 2B_1(m_i, m_\psi, m_\phi)\right], \tag{2.1}$$

where the Veltman-Passarino functions (defined as in [63]) are

$$B_0(m_i, m_\psi, m_\phi) = -\int_0^1 dx \ln \frac{(1-x)m_\psi^2 + x m_\phi^2 - x(1-x)m_i^2}{Q^2} \tag{2.2}$$

$$B_1(m_i, m_\psi, m_\phi) = -\int_0^1 dx\, x \ln \frac{(1-x)m_\psi^2 + x m_\phi^2 - x(1-x)m_i^2}{Q^2}, \tag{2.3}$$

with m_ψ the mass of the fermion in the loop, Q is the renormalization scale, and m_ϕ is the mass of the boson in the loop (vector or scalar). Throughout most of this paper, except where noted otherwise, we work in the $\overline{DR}$ scheme. For the gluino, $m_\phi = 0$ (the bosons in the loop are massless gluons), and we can evaluate the integrals directly, yielding

$$\left(\frac{\Delta m_3}{m_3}\right)_{gauge} = \frac{\alpha_3}{4\pi} C(G_3) \left(5 + 3 \ln \frac{Q^2}{m_3^2}\right). \tag{2.4}$$

For the wino, we must use the full B functions, since m_W is of order m_2. The amount of diagrams differs for the neutral and charged wino (in fact, this will be the main source of the splitting between the lightest chargino and the LSP as described later). Here we use the empirical fit of [63] for the neutralino result.

2.2.2 Matter Loops

Next, we consider the contributions of the matter content to the gaugino masses. We assume that all squarks are degenerate with mass $m_{\tilde{q}}$, and that all sleptons are degenerate with mass $m_{\tilde{l}}$, with $m_{\tilde{q}}, m_{\tilde{l}} > m_3$ (this will be important when discussing the phenomenology). In addition, we approximate the fermion masses as zero. For the gluino, only quark/squark loops contribute, giving

$$\left(\frac{\Delta m_3}{m_3}\right)_{matter} = -12\frac{\alpha_3}{4\pi}B_1(m_3, 0, m_{\tilde{q}}), \tag{2.5}$$

$$-B_1(m_3, 0, m_{\tilde{q}}) = -\frac{1}{2}\ln\frac{Q^2}{m_3^2} + I, \tag{2.6}$$

$$I = \int_0^1 dx\, x \ln[rx - x(1-x)], \quad r = \left(\frac{m_{\tilde{q}}}{m_3}\right)^2. \tag{2.7}$$

It is straightforward to evaluate I

$$I = \frac{1}{2}\left(-2 + r + (r-1)^2 \ln|1-r| - (r-2)r\ln r\right). \tag{2.8}$$

Since the top mass is fairly large, we should examine whether neglecting it is a good approximation. Including the top mass gives an extra contribution $\frac{\alpha_3}{4\pi}\frac{m_t}{m_3}f(m_{\tilde{t}_1}, m_{\tilde{t}_2})$, where f is a function of the stop masses that is ~ 1. Since in our case $m_3 \sim 1$ TeV, this term is indeed suppressed.

For the wino and bino, we divide the matter contributions into fermion/sfermion, chargino/charged Higgs, and neutralino/neutral Higgs pieces. The sfermion piece is

$$\left(\frac{\Delta m_i}{m_i}\right)_{sfermion} = -2\frac{\alpha_i}{4\pi}S(R_\Phi)B_1(m_i, 0, m_\phi), \tag{2.9}$$

summing over all chiral supermultiplets $\Phi = (\phi, \psi)$ that couple to the wino/bino. To simplify B_1, we can take the wino/bino mass to be approximately zero. At first sight this doesn't seem to be valid, since the gaugino and sfermion masses are both of roughly the same order. However, since B_1 is multiplied by α_1 or α_2, which are both very small, any error will become negligible. We have in fact checked that this is so numerically. With this approximation we find

$$-B_1(0,0,m_\phi) \quad = \quad -\frac{1}{2}\left(\ln\frac{Q^2}{m_i^2} - \ln\frac{m_\phi^2}{m_i^2} + \frac{1}{2}\right). \tag{2.10}$$

The Higgs contribution is the same for both the wino and bino. For simplicity, we take the mass of the lightest Higgs boson to be zero in the B functions (we do the same for the wino/bino again) and assume $m_H = m_{H+} = m_A$:

$$\left(\frac{\Delta m_i}{m_i}\right)_{Higgs} = -\frac{\alpha_i}{4\pi}\left[B_1(0,\mu,m_A) + B_1(0,\mu,0) + \frac{\mu}{m_i}\sin 2\beta\left(B_0(0,\mu,m_A) - B_0(0,\mu,0)\right)\right] \tag{2.11}$$

Evaluating the B functions, we find

$$B_1(0,\mu,0) \quad = \quad \frac{1}{2}\left(\ln\frac{Q^2}{m_i^2} - \ln\frac{\mu^2}{m_i^2} + \frac{3}{2}\right) \tag{2.12}$$

$$B_1(0,\mu,m_A) \quad = \quad \frac{1}{2}\left[\ln\frac{Q^2}{m_i^2} - \ln\frac{m_A^2}{m_i^2} + \frac{1}{2} + h(m_A^2/\mu^2)\right] \tag{2.13}$$

where $h(x) = \frac{1}{1-x}\left(1 + \frac{\ln x}{1-x}\right)$. Note that $|h(x)|$ is a monotonically decreasing function of x. To estimate its maximum value, recall that $m_A^2 = 2B\mu/\sin 2\beta$, so taking $B\mu \sim \mu^2$ and setting $\tan\beta = 1$ implies $x = 2$, where $|h(x)| \sim 0.3$. This is already small, and its effect on the spectrum is negligible. To keep consistent with the existing literature, we drop this term. Also,

$$
\begin{aligned}
B_0(0,\mu,0) &= \ln\frac{Q^2}{m_i^2} - \ln\frac{\mu^2}{m_i^2} + 1 && (2.14)\\
B_0(0,\mu,m_A) &= \ln\frac{Q^2}{m_i^2} - \ln\frac{m_A^2}{m_i^2} + \frac{\mu^2}{\mu^2-m_A^2}\ln\frac{m_A^2}{\mu^2} + 1. && (2.15)
\end{aligned}
$$

2.2.3 NLO Formulae

Adding in the two-loop beta function contribution to m_3, we arrive at the full NLO result

$$
\begin{aligned}
M_3 &= m_3(Q)\left(1 + \frac{3\alpha_3}{4\pi}\left[\ln\frac{Q^2}{m_3^2} + f(r) - \frac{14}{9}\right] + \frac{3\alpha_t}{2\pi}\right) && (2.16)\\
f(r) &= 1 + 2r + 2(r-1)^2\ln|1-r| + 2r(2-r)\ln r. && (2.17)
\end{aligned}
$$

The NLO bino mass is

$$
\begin{aligned}
M_1 = m_1(Q)\Bigg(1 + \frac{\alpha_1}{8\pi}\Bigg[&-22\ln\frac{Q^2}{m_1^2} + 11\ln\frac{m_{\tilde q}^2}{m_1^2} + 9\ln\frac{m_{\tilde l}^2}{m_1^2} + \ln\frac{\mu^2}{m_1^2} + \ln\frac{m_A^2}{m_1^2}\\
&+ \frac{2\mu}{m_1}\sin 2\beta\frac{m_A^2}{\mu^2-m_A^2}\ln\frac{\mu^2}{m_A^2} - 12\Bigg] + \frac{22\alpha_3}{33\pi} - \frac{13\alpha_t}{66\pi}\Bigg) && (2.18)
\end{aligned}
$$

and the NLO wino mass is

$$\begin{aligned} M_2 &= m_2(Q)\left(1+\frac{\alpha_2}{8\pi}\left[-2\ln\frac{Q^2}{m_2^2}+9\ln\frac{m_{\tilde{q}}^2}{m_2^2}+3\ln\frac{m_{\tilde{l}}^2}{m_2^2}+\ln\frac{\mu^2}{m_2^2}+\ln\frac{m_A^2}{m_2^2}\right.\right. \\ &\quad \left.+\frac{2\mu}{m_2}\sin 2\beta\frac{m_A^2}{\mu^2-m_A^2}\ln\frac{\mu^2}{m_A^2}+1.2+4.32\ln\left(\frac{m_2}{m_W}-0.8\right)\right] \\ &\quad \left.+\frac{6\alpha_3}{\pi}-\frac{3\alpha_t}{2\pi}\right). \end{aligned} \tag{2.19}$$

Notice that while our expression for the NLO gluino mass agrees with [62], there is a difference in the wino and bino masses, most notably in the coefficients of the $\ln Q^2$ terms. It seems that the authors of [62] did not include a part of the Higgsino/Higgs loop contribution in case of the bino, and omitted gauge boson loop contributions to the wino. Although numerically small, these are required for theoretical consistency, i.e. so that $\frac{dM}{d\ln Q}=0$ at one-loop order, after plugging in for the one-loop running of the gauge couplings. For the sake of completeness, we have also included a two loop α_2 contribution to the wino mass, which although it is small, should not in principle be dropped because it is of roughly the same size as the other terms.

Finally, for the case of a wino LSP, we consider the splitting between the lightest chargino, i.e. $\widetilde{W^{\pm}}$, and the LSP, $\widetilde{W^0}$, which is important for understanding the phenomenology of the gluino cascade decay. Since the tree-level splitting due to mixing in the neutralino and chargino mass matrices is small for moderate to large μ, the dominant contribution turns out to be due to gauge boson loops:

$$\Delta m_{\tilde{\chi}} = m_{\tilde{\chi}^+} - m_{\tilde{\chi}^0} = \frac{\alpha_2}{2\pi}\left[-2B_0(m_2,m_2,m_W)+B_1(m_2,m_2,m_W)\right]. \tag{2.20}$$

This is more conveniently expressed as

$$\Delta m_{\widetilde{\chi}} = \frac{\alpha_2 m_2}{4\pi}\left[f(r_W) - \cos^2\theta_W f(r_Z) - \sin^2\theta_W f(0)\right], \tag{2.21}$$

$$f(r_i) = \int_0^1 dx\,(2+2x)\ln[x^2 + (1-x)r_i^2], \qquad r_i = \frac{m_i}{m_2}. \tag{2.22}$$

The splitting is roughly independent of m_2: For $m_2 = 260$ GeV, $\Delta m_{\widetilde{\chi}} = 167$ MeV, while for $m_2 = 2.6$ TeV, $\Delta m_{\widetilde{\chi}} = 172$ MeV.

2.3 Messengers and Sleptons

To solve the tachyonic slepton problem, we introduce vector-like messenger fields in complete representations of $\mathbf{5} + \bar{\mathbf{5}}$, which get masses from the following tree-level Kähler potential terms:

$$\lambda \int d^4\theta \frac{\phi^\dagger}{\phi}\mathbf{5}\bar{\mathbf{5}} + \kappa \int d^4\theta \frac{X^\dagger X(\mathbf{5}^\dagger\mathbf{5} + \bar{\mathbf{5}}^\dagger\bar{\mathbf{5}})}{M_*^2} \tag{2.23}$$

where ϕ is the conformal compensator, X is the hidden sector field that breaks SUSY, and M_* is a UV cutoff that is naturally the Planck scale in our model. In general we consider N such sets of messenger fields, where we need $N \leq 4$ to preserve gauge coupling unification. Unification also works with one set of $\mathbf{5} + \bar{\mathbf{5}}$ and one set of $\mathbf{10} + \overline{\mathbf{10}}$ (we retain gauge coupling perturbativity in this case because the messengers have masses above 5 TeV [64]). In fact, this is what we need for our complete model with the μ problem solution, to be described below. Since a $\mathbf{10}$ has a Dynkin index of 3/2, this gives the same contribution to soft masses as a model with $N = 4$, as is clear from the formulae below.

Since it would be overly contrived to now introduce another scale in addition to $m_{3/2}$, we give the messengers masses of this order in a simple way through the EAM approach outlined in [58], which uses the Giudice-Masiero term [12]. When supersymmetry is broken, the compensator acquires an F term VEV so that $\phi = 1 + m_{3/2}\theta^2$. The messengers also get soft masses through the second term in Eq. (2.23). Since $m_{3/2} \sim F_X/M_{Pl}$, the soft masses are also set by $m_{3/2}$. We parametrize the soft mass in terms of the supersymmetric mass M as

$$m^2_{soft} = -c^2 M^2, \tag{2.24}$$

where the reason for the minus sign will become clear in a moment. To prevent the breaking of $SU(3)$, we require $c \leq \sqrt{1 - 1/\lambda}$, with $\lambda \geq 1$. For simplicity, we assume the same GM coupling for all generations of messengers and equal soft masses for each messenger pair. Generalizing these assumptions does not significantly change the picture. We therefore have a hybrid theory of gauge and anomaly mediation, with messenger scale $M = \lambda m_{3/2}$, and $F = -\lambda m^2_{3/2}$. The scalar-messenger mass matrix is

$$M^2 \begin{pmatrix} \mathbf{5}^\dagger & \bar{\mathbf{5}} \end{pmatrix} \begin{pmatrix} 1 - c^2 & -F/M^2 \\ -F/M^2 & 1 - c^2 \end{pmatrix} \begin{pmatrix} \mathbf{5} \\ \bar{\mathbf{5}}^\dagger \end{pmatrix} \tag{2.25}$$

In principle, one should also include contributions to F coming from contact terms with X. However, for the sake of simplicity, we assume that the only contribution is from the GM term, which captures the qualitative features of the general corrections. We examine the effect of the messengers on the gaugino spectrum in the next section.

Here we focus on the soft masses, which give rise to Poppitz-Trivedi D-type gauge mediation. This mechanism was used to solve the tachyonic slepton problem in [54,55, 57]. The idea is simple: since the scalars and messengers share gauge interactions, soft masses for the messengers will induce scalar masses. This contribution was calculated by Poppitz and Trivedi [56] to be

$$\Delta m_i^2 = -\sum_a \frac{g_a^4}{128\pi^4} S_M\, C_{ai}\, Str M_{mess}^2 \log\frac{\Lambda^2}{m_{IR}^2}, \tag{2.26}$$

where S_M is the Dynkin index of the messenger field, C_{ai} is the quadratic Casimir, and Λ, m_{IR} are UV and infrared cutoffs, respectively. The logarithm is large, since we take Λ at the GUT scale and $m_{IR} = M$ (The natural cutoff for our model is the Planck scale, so this is not the entire contribution. There is also a correction coming from physics between the GUT and Planck scales that is not log-enhanced, and includes unknown threshold corrections at the GUT scale). As pointed out in [55], the large logs can be resummed using the one-loop RGE's for the gauge couplings, yielding

$$\Delta m_i^2 = -\sum_a \frac{S_M\, C_{ai}\, Str M_{mess}^2 [g_a^2(\Lambda) - g_a^2(m_{IR})]}{8\pi^2 b_a}, \tag{2.27}$$

with b_a the β function coefficient above the messenger scale. In our case, $StrM^2_{mess} = 4m^2_{soft}$, so we need m^2_{soft} to be negative in order to get a positive contribution, hence the minus sign in the definition above. Since $\Delta m_i^2 \sim (\Delta g^2/16\pi^2)(c\lambda)^2 m^2_{3/2}N$, it is clear that this contribution can easily be as large as that from AMSB for relatively small values of $c\lambda$, i.e. they need not even be $O(1)$, pushing the slepton masses positive at the messenger scale.

2.4 Gaugino Spectrum

We now take a closer look at how the messengers change the gaugino spectrum. Integrating out the messengers takes the soft terms off of the anomaly mediated trajectory. This means that to get the gaugino masses at the weak scale, we need to compute them first at the messenger scale and then run down. We first run up the gauge couplings at two loops to do this. Immediately above the messenger scale the gaugino masses are on the anomaly-mediated trajectory. Using the two-loop beta functions, which include contributions from the messengers, we find

$$\begin{aligned} m_1(M) &= \frac{\alpha_1}{4\pi}\left(11+\frac{5}{3}N+\frac{\alpha_2}{4\pi}(3N+9)+\frac{\alpha_3}{4\pi}\left(\frac{32}{9}N+\frac{88}{3}\right)-\frac{13\alpha_t}{6\pi}\right)m_{3/2} \\ m_2(M) &= \frac{\alpha_2}{4\pi}\left(1+N+\frac{\alpha_2}{4\pi}(7N+25)+\frac{24\alpha_3}{4\pi}-\frac{3\alpha_t}{2\pi}\right)m_{3/2} \\ m_3(M) &= \frac{\alpha_3}{4\pi}\left(-3+N+\frac{9\alpha_2}{4\pi}+\frac{\alpha_3}{4\pi}\left(\frac{34}{3}N+14\right)-\frac{\alpha_t}{\pi}\right)m_{3/2}. \end{aligned} \tag{2.28}$$

We work to two-loop order because there is the possibility of near degeneracy of gaugino masses and we are interested in very small gluino-LSP mass splittings, so corrections at the percent level are important. Since $\alpha_3 \sim 3\alpha_2$ at the weak scale, we see that we can make the gluino and wino masses approximately equal at the messenger scale by choosing $N = 2$. There are also threshold corrections from integrating out the messengers, and the exact expression depends on whether or not the messengers have soft masses. Here we first consider the more general case of messenger soft masses, which are needed in our model. We then discuss the simpler case with no soft masses.

2.4.1 Soft Masses

Adapting the formula in [56], we find

$$\Delta m_i = \frac{\alpha_i}{2\pi} f_t(y_1, y_2) N m_{3/2}, \quad f_t(y_1, y_2) = \frac{y_1 \log y_1 - y_2 \log y_2 - y_1 y_2 \log(y_1/y_2)}{(y_1 - 1)(y_2 - 1)(y_2 - y_1)} \tag{2.29}$$

with $y_1 = M_1^2/M^2$, $y_2 = M_2^2/M^2$, where $M_{1,2}^2$ are the eigenvalues of the scalar messenger mass-squared matrix (we adopt a convention where M_1 is the larger of the two). For the simplest case of universal GM couplings and soft masses that we consider

$$y_1 = 1 - c^2 + 1/\lambda, \qquad y_2 = 1 - c^2 - 1/\lambda. \tag{2.30}$$

For λ not too close to 1 and small soft masses, $f_t \sim 0.5$. As $\lambda \to 1$ (soft masses

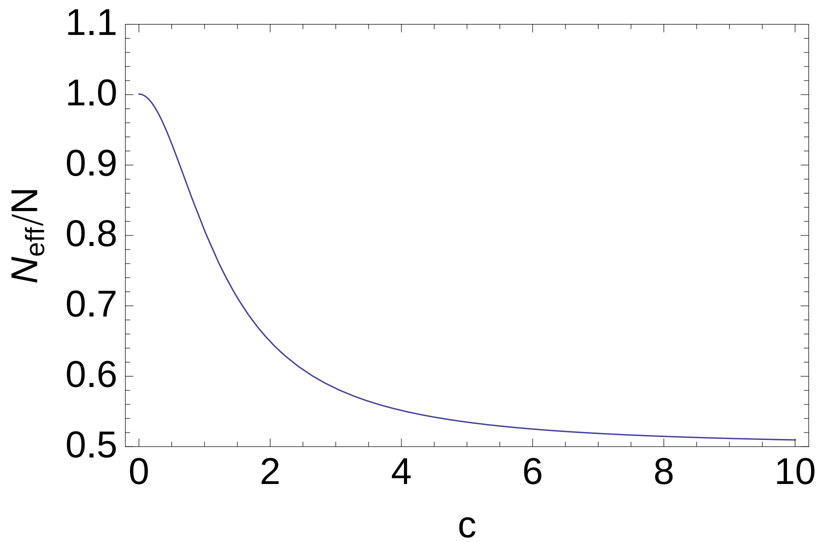


Figure 2.1: Effect of c for positive m^2_{soft}

still small), $f_t \to 0.7$ since there is an enhancement due to contributions from higher order terms in F/M^2 [65]. To examine the role of the soft masses, it is useful to consider the effective number of messengers N_{eff}, a continuous variable defined by

$$N_{eff} = \frac{1+2f_t}{2}N. \tag{2.31}$$

Here we consider both positive and negative m^2_{soft} for λ not too close to 1. In the positive case, as can be seen in Fig. 2.1, N_{eff} decreases with increasing c. As c becomes bigger the soft mass dominates over the supersymmetric mass and $N_{eff} \to N/2$. Note that with positive m^2_{soft}, c is not constrained to be smaller than one. N_{eff} increases with c for negative m^2_{soft}, although it is very gradual until $c \sim 0.6$, as shown in Fig. 2.2.

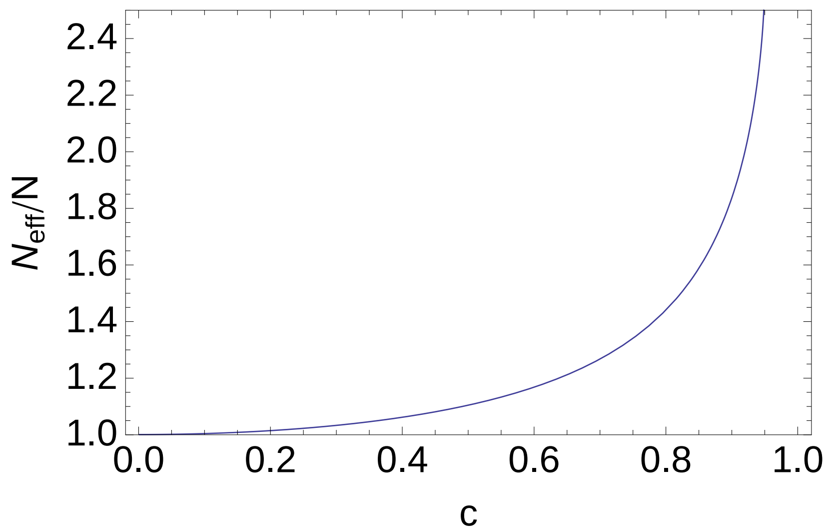


Figure 2.2: Effect of c for negative m^2_{soft}

2.4.2 No Soft Masses

The compression of the gaugino spectrum due to messengers is an interesting aspect of extended AMSB theories that, according to our knowledge, has not been investigated previously. We therefore consider it now as an independent module that can be incorporated into other models, i.e. we just focus on the gaugino masses. In this case there is no reason to keep the messenger soft masses, so we simplify the setup by just keeping the GM term for the messengers. In this case, the threshold correction from integrating out the messengers takes the simpler form [60]

$$\Delta m_i(M) = -\frac{1}{g_i}(\beta_i' - \beta_i)G(F/M^2)\frac{F}{M}, \tag{2.32}$$

where β' is the beta function above the messenger threshold, β is the beta function below the messenger threshold, and G is the enhancement factor mentioned above. G(x) increases monotonically from 1 to 1.386 as x goes from 0 to 1 [65]. Here we also take different couplings λ_D and λ_T for the triplets and doublets, although the coupling cancels out in F/M, which means that we can only adjust the value of the higher order contribution G separately for the triplets and doublets. Explicitly, the threshold corrections to two-loop order are

$$\begin{aligned}
\Delta m_1 &= \frac{\alpha_1}{4\pi}\left(\frac{5}{3}+\frac{3\alpha_2}{4\pi}+\frac{8\alpha_3}{9\pi}\right)\left[G(x_D)+\frac{2}{3}G(x_T)\right]\frac{3}{5}Nm_{3/2} &\quad (2.33)\\
\Delta m_2 &= \frac{\alpha_2}{4\pi}\left(1+\frac{7\alpha_2}{4\pi}\right)G(x_D)Nm_{3/2} &\quad (2.34)\\
\Delta m_3 &= \frac{\alpha_3}{4\pi}\left(1+\frac{17\alpha_3}{6\pi}\right)G(x_T)Nm_{3/2}, &\quad (2.35)
\end{aligned}$$

where $x = -1/\lambda$.

We then run down the gaugino masses to 1 TeV using the one-loop RGEs (we include the next-to-leading order correction for the gluino) and compute the pole masses by adding the corrections appearing in the square brackets in Eqs. (2.16), (2.18), and (2.19). To keep the log term from getting too large, we compute the gluino pole mass at a separate scale, equal to the running mass at 1 TeV. Here we use the $\overline{MS}$ equations so the pole mass equation for the gluino is modified to

$$M_3 = m_3(Q)\left(1+\frac{3\alpha_3}{4\pi}\left[\ln\frac{Q^2}{m_3^2}+f(r)-1\right]\right). \quad (2.36)$$

Finally, to calculate the splitting between the gluino and the LSP, we diagonalize

N	$m_{3/2}$ (TeV)	λ_D	λ_T	M_1 (GeV)	M_2 (GeV)	M_3 (GeV)	ΔM (GeV)
1	70	2.5	2.5	851	618	523	$\tilde{g}$ lightest
2	40	2.5	1.2	608	580	650	74
3	40	1.5	4.0	721	818	1130	411
4	34	1.5	4.0	712	888	1439	728

Table 2.1: Gaugino spectra

the neutralino mass matrix using the calculated pole masses for the bino and wino.

We now discuss the gaugino spectra for different numbers of messengers. Examples are presented in Table 2.1, with M_3 the gluino mass and ΔM the gluino-LSP splitting. We assume that the scalars and the Higgsinos are somewhat heavier than the gluino; here we choose an arbitrary mass of 1.6 TeV for all, and take $\tan\beta = 5$. With heavy squarks, the dominant SUSY production mechanism at the LHC is gluino pair production. Each gluino then eventually decays to the bino or wino LSP. In either case, direct decay to the LSP, $\widetilde{g} \to jj\widetilde{\chi}^0$, is possible and is the dominant mode for a bino LSP. For the case of a wino LSP, there is also cascade decay through the charged wino, $\widetilde{g} \to jj\widetilde{\chi}^{\pm}$. For $\Delta m_{\widetilde{\chi}} > m_\pi$, $\widetilde{\chi}^{\pm} \to \pi^{\pm}\widetilde{\chi}^0$ happens 98% of the time. For the range of wino masses that we consider $\Delta m_{\widetilde{\chi}}$ is roughly 170 MeV, so this mode is always open. Although the charged wino can travel a macroscopic distance before decaying [62] (about 1 cm in our case), a displaced vertex analysis is not possible because the pion is too soft. Thus, in terms of observable signatures, we can simply describe the decay to a wino LSP as also being direct.

As pointed out in [66], despite the large production cross-section, these events are difficult to detect when the gluino is nearly degenerate with the LSP since the jets from the decay are very soft. Furthermore, these events may not even have the large E_T^{miss} that is usually a hallmark of R-parity conserving theories, meaning that they will be hidden in QCD background. Even if the gluinos are strongly boosted the LSP momenta will approximately cancel unless the gluino momenta are unbalanced by the emission of initial or final state radiation.

For $N = 1$, we do not obtain an acceptable spectrum–the gluino is always the lightest. This is because in this case the contributions to the gluino mass from anomaly mediation and the messenger threshold correction have opposite sign. Although the correction due to squarks is sizable, bumping up the gluino substantially would require very heavy squarks. This is because the squark correction increases slowly as the ratio of the squark mass to m_3 is raised, as can be seen in Fig. 2.3.

For $N = 2$, the wino is the LSP unless λ_D gets very close to 1, in which case it is the bino. The spectrum is very compressed, with ΔM no bigger than about 80 GeV. As can be seen in Fig. 2.4, ΔM increases with λ_D because the messenger threshold correction to the wino gets smaller, decreasing its mass. Conversely, decreasing λ_D raises the wino mass more than the bino mass because of the smallness of α_1, eventually pushing the wino above the bino for $\lambda_D \sim 1$.

The $N = 3$ and $N = 4$ cases are not as compressed but are still squeezed sub-

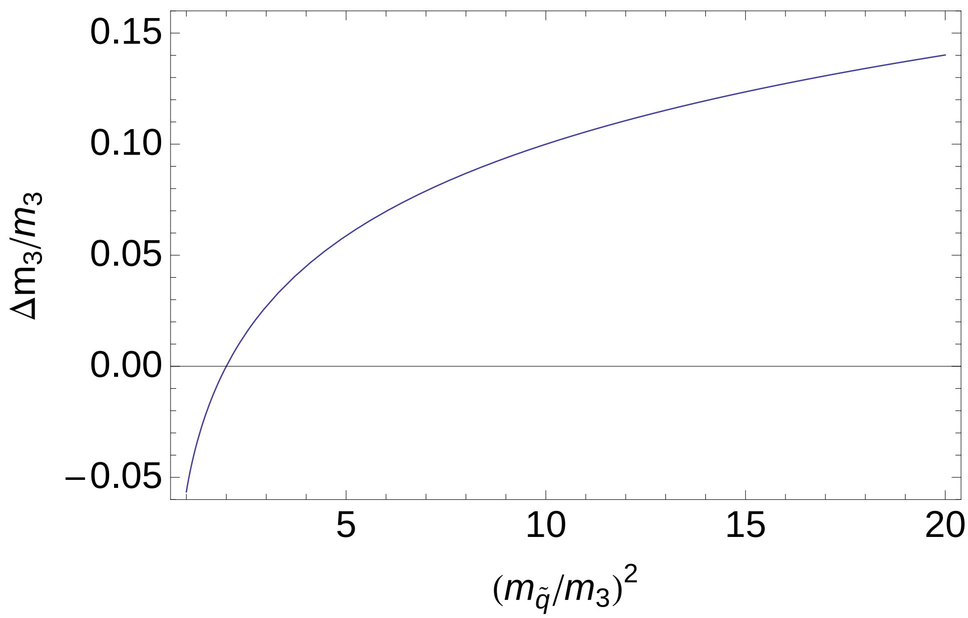


Figure 2.3: Squark contribution to gluino mass

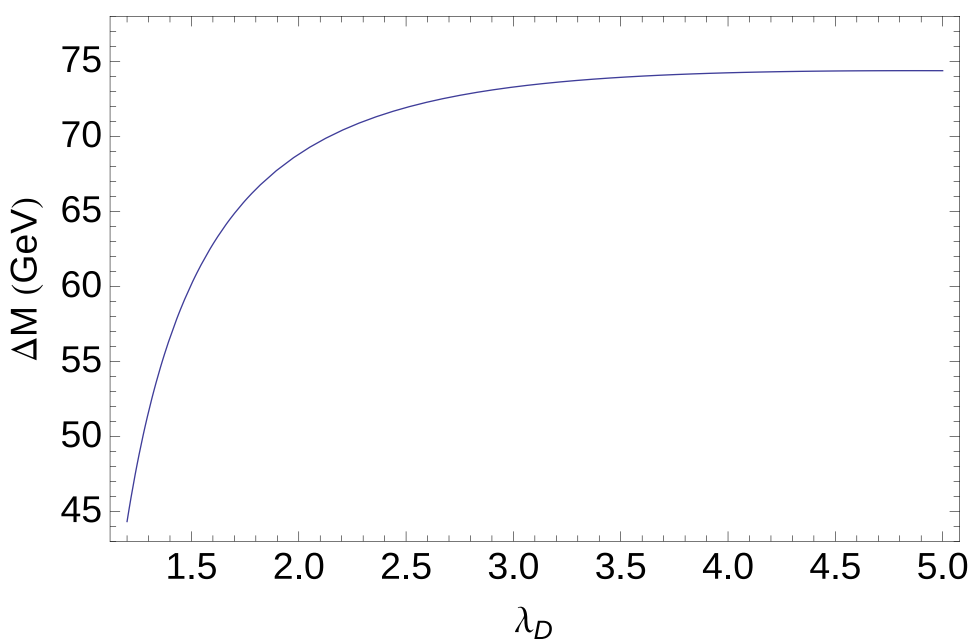


Figure 2.4: Gluino-LSP Splitting for $N = 2$

stantially compared to pure AMSB. The bino is the LSP in both cases. We focus on gluino masses below 1.5 TeV, since this is the region that many claim has already been ruled out. Varying λ_D has only a slight effect because ΔM is a few hundred GeV whereas Δm_1 is about 200 GeV, meaning that ΔM can only change by a few tens of GeV.

In summary, for 2, 3, or 4 messenger pairs, relatively light gluinos in the range of 600 to 1400 GeV are still viable. Additionally, compression of the gaugino spectrum is attractive in Split Supersymmetry, where the gluino is perhaps the only sign of new physics and is out of the reach of the LHC with the usual AMSB mass hierarchy and wino/bino thermal dark matter. A detailed discussion about messengers in Split SUSY can be found in Ch. 3.

2.5 Complete Example Spectra

In the previous sections we have discussed separately the solution to the tachyonic slepton problem through D-type gauge mediation and the compression of the gaugino spectrum from the messenger threshold in EAM. We now combine the two to produce complete spectra for different numbers of messengers in the theory. These are listed in Table 2.2. We assume that a solution to the μ problem exists (more on this in the next section) and take $\mu = 1$ TeV (with $B\mu \sim \mu^2$) and three values of $\tan\beta$. For

		$N = 2$	$N = 3$	$N = 4$
inputs:	$m_{3/2}$	40000	40000	30000
	λ	1.3	2.5	3.0
	c	0.25	0.10	0.09
	$\tan\beta$	10.0	13.0	3.0
	μ	986	984	979
sleptons:	$m_{\tilde{e}_L}$	750	762	975
	$m_{\tilde{e}_R}$	657	754	669
	$m_{\tilde{\nu}_L}$	750	762	975
squarks:	$m_{\tilde{u}_L}$	1880	1959	2254
	$m_{\tilde{u}_R}$	1704	1752	2079
	$m_{\tilde{d}_L}$	1880	1959	2254
	$m_{\tilde{d}_R}$	1732	1805	2060
stops:	$m_{\tilde{t}_1}$	1532	1565	1884
	$m_{\tilde{t}_2}$	1816	1889	2177
gauginos:	$m_{\tilde{B}}$	619	714	645
	$m_{\tilde{W}}$	616	795	795
	$m_{\tilde{g}}$	703	1193	1439
Higgs sector:	m_A	5727	5715	3417

Table 2.2: Example spectra for $N = 2, 3, 4$. All masses are in GeV.

these chosen parameters, we calculate the additional contributions to the Higgs soft masses that are needed for proper EWSB and add them at the messenger scale.

In the $N = 2$ and $N = 3$ cases, we choose squarks in the range of $1.7 - 1.9$ TeV, with a slightly lighter, right-handed stop (there is little mixing), and sleptons that are lighter by about a factor of two. The $N = 4$ case has slightly heavier squarks compared to these. In principle, the squarks could be much lighter in these scenarios, and sneutrinos would become the LSP (a spectrum we do not study, as we are investigating scenarios with lighter gauginos). Since the D-type GMSB contribution dominates over its AMSB counterpart – it is about an order of magnitude larger – the mass hierarchy can be explained by ratios of gauge couplings, as in minimal GMSB. However, the near equality of the slepton masses in the $N = 2$ and $N = 3$ cases is a departure from this and arises from both EWSB constraints and our choice of larger $\tan\beta$. Because μ is fairly big, we need $m_{h_d}^2$ to be large to obtain sizable values of $\tan\beta$. In addition, $m_{h_u}^2$ must be negative to satisfy the other EWSB equation involving m_Z. This means that the oft-neglected $\alpha_1^2(m_{h_u}^2 - m_{h_d}^2 + ...)$ piece in the RGE for $m_{\tilde{e}_R}^2$ is significant and drives the mass up considerably when running down. In the $N = 4$ case, we chose a smaller $\tan\beta$ so $m_{h_d}^2$ is not as large and $m_{h_u}^2$ is positive, and this effect is greatly diminished.

The gaugino masses are similar to those in Table 2.1 because the messenger soft masses have only a slight effect on the messenger threshold correction for the small values of c that we need. As noted before, we ignore the $N = 1$ case since that results in a gluino LSP. The mechanism that generates $\mu/B\mu$ will obviously have an impact on the physical Higgs mass, which does not concern us here.

2.6 Addressing the μ Problem

It is well known that one cannot write down a tree-level μ term in minimal AMSB because the resulting $B\mu$ would be a loop factor too large to allow for proper EWSB. However, D-type gauge mediation is an extra element in our model that contributes to the Higgs soft masses. Taking inspiration from [67], we examine whether we can use this extra freedom to increase the Higgs soft masses enough to make EWSB work despite the large $\mu/B\mu$ hierarchy. We find roughly that to barely satisfy the EWSB stability condition ($\tan\beta = 1$), $c\lambda \sim 3$, yielding squark masses close to 15 TeV, which is clearly not acceptable. Although the two Higgs doublets can be considered a "messenger pair", a GM term is not an option; since μ is large, obtaining a light Higgs would require fine tuning. It is clear that we must add something new to our model.

We can try to solve the μ problem by taking the simplest extension, one that was considered in the early days of gauge mediation [68]. We generate $\mu/B\mu$ by coupling the messengers to the Higgs doublets in the following way:

$$W \supset z_u \overline{\mathbf{10}}\,\mathbf{5} H_u + z_d \mathbf{10}\,\bar{\mathbf{5}} H_d. \tag{2.37}$$

This again leads to the same $\mu/B\mu$ hierarchy, but also gives an extra contribution to the Higgs soft masses, so that we don't have to rely solely on the GMSB contribution. Working to all orders in F/M^2 and including the soft masses, the new yukawa couplings generate the following contributions to the Higgs sector:

$$\begin{aligned}
\mu &= \frac{z_u z_d}{2\pi^2} f_t(y_1, y_2) m_{3/2} && (2.38)\\
B\mu &= -\frac{z_u z_d}{4\pi^2} \ln(1-x^2)\lambda^2 m_{3/2}^2 && (2.39)\\
\Delta m_{h_{u,d}}^2 &= -\frac{z_{u,d}^2}{4\pi^2}\left[\left(2-c^2\right)\ln(1-x^2) + \left(1-c^2\right) x \ln\frac{1+x}{1-x}\right]\lambda^2 m_{3/2}^2, && (2.40)\\
x &= \frac{1}{\lambda(1-c^2)}\,. && (2.41)
\end{aligned}$$

For illustrative purposes only, we also give the formulas for the soft parameters to lowest order in $1/\lambda^2$ and c^2. These should roughly show the correct qualitative behavior since $c < 1$ and $\lambda \geq 1$:

$$\begin{aligned}
B\mu &= \frac{z_u z_d}{4\pi^2}(1+2c^2) m_{3/2}^2 && (2.42)\\
\Delta m_{h_{u,d}}^2 &= \frac{z_{u,d}^2}{4\pi^2} c^2 m_{3/2}^2. && (2.43)
\end{aligned}$$

As first noted in [68], it is clear from the above that the Higgs soft masses do not get a contribution at lowest order with messenger soft masses equal to zero. Because of this fact, we must introduce a hierarchy between z_u and z_d so that $\tan\beta$ is not fixed too closely to one, i.e. we need $m_h^2 > B\mu$ for one of the Higgs doublets. So the EWSB will be achieved using the approach outlined in [67], with a large $\mu/B\mu$ hierarchy and a smaller one between $B\mu$ and either $m^2_{h_u}$ or $m^2_{h_d}$. Taking this to be h_u, we find that $\sin 2\beta \sim 2(z_d/z_u)$. Since we do not want Higgsinos that are too light, we must fix the product $z_u z_d$. However, we cannot make z_u larger than 1 without hitting a Landau pole before the GUT scale. These constraints imply that μ is small, about 300 GeV. The gluino mass with a scalar spectrum similar to that of the $N = 2$ or $N = 3$ cases in Table 2 is then about 1.5 TeV. Although this spectrum is viable (barely), it is only because of the size of the gluino mass and not because of compression, so we do not find this attempt at a μ problem solution to be satisfactory in this minimal form. Note also that we need to tune the value of the other yukawa coupling so that the condition for EWSB in this case, $B\mu^2 > m^2_{h_u} m^2_{h_d}$, nearly becomes an equality. This has to be done so that we obtain the experimental value of m_Z in the other EWSB equation.

A further issue is the physical Higgs mass. In our model with all messenger soft masses negative, the scalars in each messenger multiplet are lighter than the fermions, so the new Higgs couplings will produce a negative contribution to the Higgs mass squared, whereas to achieve a Higgs mass of 126 GeV we need a large, positive correction for our case of $\sim$ 2 TeV stops [19]. It seems this can be overcome by choosing different soft masses for the **5** and **10** multiplets, with the scalar soft masses of the **10** negative so that the D-type GMSB contribution to the MSSM scalar masses is still positive. Analogously to the top/stop contribution, the messenger contribution to the Higgs mass depends on the logarithm of the ratio of the geometric average of the scalar masses to the fermion mass. So if the scalars from the **5**'s have positive soft masses that are larger in magnitude than those of the scalars from the **10**'s, the average messenger scalar mass can be made larger than the fermion mass, yielding a positive contribution to the Higgs mass.

To obtain a viable solution, we clearly need an additional contribution to μ. This can be done by introducing a singlet and working in the context of the NMSSM. This has been investigated recently in the context of GMSB with new messenger couplings in [69]. In this case the physical Higgs mass results from the large A terms that are produced in this model. Since the singlet is not charged under the SM gauge groups, it does not affect the AMSB or GMSB contributions to the soft masses. In particular, the gaugino spectrum remains unchanged. This model could then eventually be realized in a 5D brane-world setup similar to that of [54, 55], where the visible (MSSM) and hidden sectors are localized on different branes, with the gauge fields and the messengers in the bulk.

2.7 Conclusions

The most recent data from the LHC excludes gluinos with masses less than ~ 1.5 TeV in typical models that have a significant gluino-LSP mass splitting, putting a strain on naturalness. However, gluinos as light as 550 GeV are still allowed for very small mass splittings. We have presented a simple and novel way that such a compressed gaugino spectrum occurs naturally in the context of AMSB.

AMSB models typically have spectra that feature a large spread in the gaugino masses − the gluino is almost 10 times heavier than the wino LSP. Such models can be out of the reach of the LHC for LSP masses of $\mathcal{O}(1 \text{ TeV})$. We find that the presence of messengers in the AMSB framework compresses the spectrum. The resulting spectra have a relatively light gluino with a mass in the range of 600 to 1400 GeV that is no heavier than about twice the LSP mass, with the exact values dependent on the number of messengers N used. For the $N = 1$ case, the gluino is the LSP, while the $N = 2$ case yields a wino or bino LSP depending on the value of the coupling in the Giudice-Masiero term for the messenger doublets, and the mass splitting between the gluino and the LSP is of the order of tens of GeV. The $N = 3, 4$ cases are less compressed and yield a bino LSP.

We have provided expressions for the gaugino pole masses which differ from the expressions present in the literature for the case of the wino and bino. We would like to emphasize that we have confidence in our expressions being correct as the pole masses are independent of the running scale in our case. We have discussed in detail the steps to obtain the pole masses to account for the differences.

Apart from compressing the spectrum, the messengers are crucial in building a complete phenomenological model without singlets, as they help solve the tachyonic slepton problem in AMSB in a way previously suggested in the literature. Contact terms between the messenger fields and hidden sector SUSY breaking result in soft masses for the messengers. Through Poppitz-Trivedi D-type gauge mediation, these soft masses generate contributions to scalar masses which are positive if the soft masses squared are taken to be negative. This contribution is of the same order or greater than the AMSB contribution and thus solves the tachyonic slepton problem. The only hurdle to a complete model that then remains is the μ problem. We have made an attempt that indicates that solutions can be found in our framework. It involves new yukawa couplings between the Higgs doublets and messengers (i.e. not new fields). However, the minimal version that we considered is deficient, since it produces a light Higgsino LSP and requires considerable fine-tuning. We leave open for future work possible extensions with both positive and negative messenger soft masses and/or the NMSSM as an avenue for resolving the remaining problems.

Chapter 3

Simply Unnatural (Mini-Split) Supersymmetry

3.1 Introduction

Here we describe and explore the simplest picture of the the world arising from fine-tuned supersymmetric theories. Our guiding principle is that the model should be "simply un-natural". There is an explicit, un-natural tuning for the weak scale with a clear "environmental" purpose, but in every other way the theoretical structure should be as simple as possible. To this end, we will follow where the theory leads us, without any clever model-building gymnastics. Following what theories of supersymmetry breaking "want to do" leads us to theories with a "minimally split" spectrum where gauginos are near 1 TeV, while scalars, Higgsinos, and the gravitino are parametrically heavier by a loop factor, at a scale m_{sc} between $\sim 10^2 - 10^3$ TeV. This kind of spectrum has long been a ubiquitous feature of simple, concrete models of SUSY breaking. Its modern manifestation was in the context of theories with anomaly mediated SUSY breaking [70], *without* the clever sequestering mechanism of [71].

In [70], the heavy scalars were thought of as something of an embarrassment. This spectrum was later proposed as a serious possibility for supersymmetric theories in [20, 21, 72], put forward as the "simplest model of split SUSY" in [73], and further studied in [74]. We re-initiated a study of this scenario in [75,76]. For obvious reasons, this spectrum has received renewed attention of late [77–82]. The Higgs mass prefers this "minimally split" spectrum, rather than the more radical possibility of scalars up to around $\sim 10^{13}$ GeV [20]. This is perfectly in line with the "simply un-natural" perspective, since theories with much heavier scalars need extra theoretical structure to suppress gaugino masses by much more than a loop factor relative to the gravitino mass.

With this split spectrum, gaugino masses receive comparable contributions from anomaly mediation and the heavy Higgsinos, as well as other possible vector-like matter near the scale m_{sc}. As we will see, this picture has important consequences for flavor physics, as well as a host of novel collider signals that constrain the scale m_{sc} in an interesting way.

In Sec. 3.2, we present in more detail the Mini-Split framework that was introduced in Sec. 1.4.3, showing that it is natural to get Higgsino masses of the same order as that of the scalars, and that this is consistent with unification. There we also introduce new vector-like states and study the effects that such messengers have on unification and the gaugino spectrum, and discuss the implications for dark matter. In Sec. 3.3, we point out that a radiative model of flavor arises simply in the Mini-Split setup. This idea is developed further and explored in Ch. 4. A discussion of the experimental signals of our model is found in Sec. 3.4, and we conclude in Sec. 3.5.

3.2 Simplest Tuned Picture of the World

3.2.1 Model and Spectrum

Supersymmetry breaking must give all superpartners in the MSSM masses above their current bounds. Once supersymmetry is broken, there are no symmetries protecting the sfermion masses, and thus scalar masses are expected at some level. On the other hand, (Majorana) gaugino masses require the breaking of an R symmetry, and are thus not guaranteed to arise at the same level. In the case where supersymmetry breaking is communicated via irrelevant operators suppressed by a scale M, sfermion and gaugino masses could arise from the operators of the form

$$\int d^4\theta \frac{X^\dagger X Q^\dagger Q}{M^2} \quad \text{and} \quad \int d^2\theta \frac{Y W^\alpha W_\alpha}{M}, \tag{3.1}$$

where Q and W^α represent visible sector matter and gauge superfields respectively and X and Y are hidden sector chiral superfields which have non-zero VEVs in their auxiliary ('F') components. There are no requirements (other than the absence of a shift symmetry) on the quantum numbers of X, and thus it could be any field from the hidden sector. On the other hand, Y is required to be an exact gauge and global singlet. This stringent requirement makes it clear that gaugino masses will typically not be the same size as scalar masses for generic hidden sectors. In fact, most models of supersymmetry breaking sectors do not contain such singlets [83–90], and this affects both gravity and gauge mediation. While this problem has been 'solved', in the sense that models generating larger gaugino masses have been found [91, 92]–with non-generic superpotentials and/or many discrete symmetries imposed–we take the position that generic models of supersymmetry breaking produce much larger scalar masses than gaugino masses, that is, this is what the models *want* to do. In line with our "simply un-natural" philosophy, we assume the theory-space tuning required to have degenerate sfermions and gauginos is more severe than that required to get the correct electroweak scale.

Another contribution to superpartner masses that theories of broken supersymmetry "want to generate" arises from anomaly mediation. The breaking of R symmetry associated with tuning away the cosmological constant with a constant superpotential gives rise to gaugino masses of order a loop factor beneath the gravitino mass. While there are clever ways to suppress this contribution [20, 93], we consider this contribution generic. Thus, in gravity mediated theories (where M is approximately the Planck scale or a bit below), the gaugino masses will typically end up a loop below the scalar masses.

In Planck- or string-scale mediation of supersymmetry breaking, one possibility for removing the dominant scalar mass operator in Eq. (3.1) is through sequestering, *i.e.*, separating the visible and hidden sectors in an extra dimension or 'conformal throats' [71, 94]. There has been some debate about how generic such sequestering is ([95–98]). We will make the assumption that sequestering is not generic.

We are led to a class of gravity mediation models in which the gaugino masses are roughly a loop factor below the scalars. A possibility we will not explore is single-sector gauge mediation, where again gaugino masses tend to be much more than a loop-factor lighter than scalar masses.

In addition, for "A terms" to be at the same scale as scalar masses, they would have to be generated by operators like

$$\int d^2\theta \frac{Y H_u Q U^c}{M}, \tag{3.2}$$

again requiring a gauge singlet in the hidden sector. Thus our philosophy suggests A terms are small – again, dominated by a one-loop suppressed contribution from anomaly mediation. This will of course have an important impact on the Higgs mass predictions.

Of course the natural version of these models were ruled out when gauginos were not discovered in the 1 GeV range! If these theories are realized in Nature, some kind of "pressure" on the measure pushing towards higher supersymmetry scales is needed, which counteracts the tuning of the cosmological constant and the electroweak scale.

We will not attempt to address the notoriously ill-defined question of quantifying these pressures. We will simply assume that whatever the measure is, the likelihood of having a hidden sector that produces degenerate sfermions and gauginos is much smaller than that of a split spectrum with the obvious fine-tuning for electroweak symmetry breaking. We stress that with the spectra we are considering, the fine-tunings at the $\sim 10^{-4} \to 10^{-6}$ level are obviously very severe from the perspective of naturalness, but are dwarfed by the $10^{-60} - 10^{-120}$ levels of fine-tuning for the cosmological constant or the usual 10^{-30} level tuning for the hierarchy problem.

Higgsinos, the μ term, and the Giudice-Masiero mechanism

What about the Higgsinos? The μ term, $W \supset \mu H_u H_d$, breaks both the Peccei-Quinn (PQ) symmetry and potentially an R symmetry, and thus there can be trivial reasons why it is much smaller than the Planck scale. The simplest operator that generates a μ term is the one suggested long ago by Giudice and Masiero [12]:

$$\lambda \int d^4\theta H_u H_d \tag{3.3}$$

where λ is an arbitrary coefficient. In global (flat-space) supersymmetry, this operator represents a total derivative. When including supergravity using the conformal compensator language [99–101], one should multiply this operator by $\phi^\dagger/\phi$ (assuming conformal weights of fields corresponding to their canonical dimensions). The compensator is given by $\phi \simeq 1 + \theta^2 m_{3/2}$, where $m_{3/2}$ is the gravitino mass, as long as the theory has no Planck scale VEVs [102]. Integrating out the Higgsinos and some of the scalars will generate a gauge-mediated-like contribution to the gaugino masses at one loop. The contribution will take gauginos off of the 'anomaly-mediated trajectory' in a special way – a right-magnitude, but wrong-sign contribution to gaugino masses [58, 59]. However, the threshold correction will be affected by squared soft masses for the scalars, and is suppressed when $m_{sc}^2 > m_{3/2}^2$.

In addition, the operator itself appears highly tuned when seen from a different frame. One can remove a chiral operator $\mathcal{O}$ from the Kähler potential $\mathcal{K}$ via the transformation

$$\mathcal{K} \to \mathcal{K} - (\mathcal{O} + \mathcal{O}^\dagger) \quad \text{and} \quad W \to We^{\mathcal{O}/M_{pl}^2}. \tag{3.4}$$

For $\mathcal{O} = \lambda H_u H_d$, the term in Eq. (3.3)) becomes terms in the superpotential, $(1 + \lambda(H_u H_d/M_{pl}^2) + ...)(W_{hid} + W_0 + W_{vis})$, where W_{hid} contains the fields involved in dynamical supersymmetry breaking and W_0 is the operator that generates a constant superpotential. Thus a pure Giudice-Masiero (GM) term, Eq. (3.3)), is a result of a precise relationship between the coefficients of two operators, $H_u H_d W_{hid}$ and $H_u H_d W_0$ (up to H^2/M_{pl}^2 corrections). This particular combination could result from a direct coupling of the curvature, whereas direct couplings to the constant superpotential and supersymmetry breaking sectors could be suppressed due to sequestering. However, we are assuming that there is no sequestering, and thus we do not have a predictive relationship between the effective μ and $B\mu$ terms. For example, if only the $H_u H_d W_0$ operator existed, the threshold would be purely supersymmetric, and, in the limit of vanishing scalar soft masses, would keep gauginos on the anomaly-mediated trajectory (of the MSSM without Higgsinos).

Having said this, for the sake of simplifying parameter space, we will take the case of pure GM as the 'central value' of the threshold correction in theory space. Regardless of the details of the threshold, it is clear that it is trivial to generate $\mu \sim m_{3/2}$ in multiple ways.

Of course since the μ term also breaks PQ symmetry, it is possible to imagine that the Higgsinos are lighter, near the same scale as the gauginos, as in the earliest models of Split SUSY [20].

One can imagine suppressing these operators, as they explicitly break the PQ symmetry (under which $H_u H_d$ is multiplied by a phase). For a pure GM term, approximate PQ symmetry implies $\lambda \ll 1$ and for $m_{scalar} \sim m_{3/2}$, this leads to a suppression of $\mu = \lambda m_{3/2}$ and $B\mu = \lambda m_{3/2}^2$, and thus $\mu \sim m_{scalar}/\tan\beta$. In the limit where $\lambda \to 0$, or more generally when Planck-suppressed superpotential couplings between $H_u H_d$ and W_0 or the hidden sector are absent, $\tan\beta$ is large yet the Higgs mass requires low scalar masses, thus rendering the spectrum unviable. In principle, the Kähler potential operator $X_{hid}^\dagger X_{hid} H_u H_d$ could generate $B\mu \sim m_{scalar}^2$, in which case μ would be generated by gaugino loops such that $\mu \sim (\alpha/4\pi) m_{gaugino}$. However, we see no symmetry reason for this limit and do not explore this spectrum further (its phenomenology was explored in [78, 82]).

If $\mu^2 \gg m_{sc}^2$, then electroweak symmetry is only broken if the coefficient of the scalar bilinear $H_u H_d$ (the $B\mu$ term) approaches the limit $B\mu \to \mu^2$. This is an

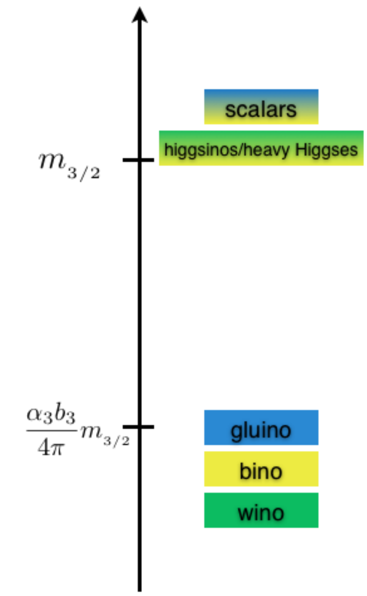


Figure 3.1: A "simply unnatural" spectrum.

interesting case, as $\tan\beta \to 1$ in this limit, a value which is disfavored in the 'natural' MSSM both because the physical Higgs mass is too low, and because the top Yukawa coupling gains a Landau pole below the GUT scale. Neither of these present a problem with heavier scalars. However, this limit requires not only the tuning for electroweak symmetry breaking ($\lambda \to 1$), but also the magic of a pure GM mass. There may not be a clear UV reason to naturally favor this point in parameter space, but it is at least interesting that it is allowed phenomenologically and should be considered.

Spectrum and Unification

For our minimal model, we take scalar masses and Higgsinos to be roughly degenerate $m_{sc} \sim \mu \sim m_{3/2}$ (Figure 3.1), and determine the overall scale favored by a

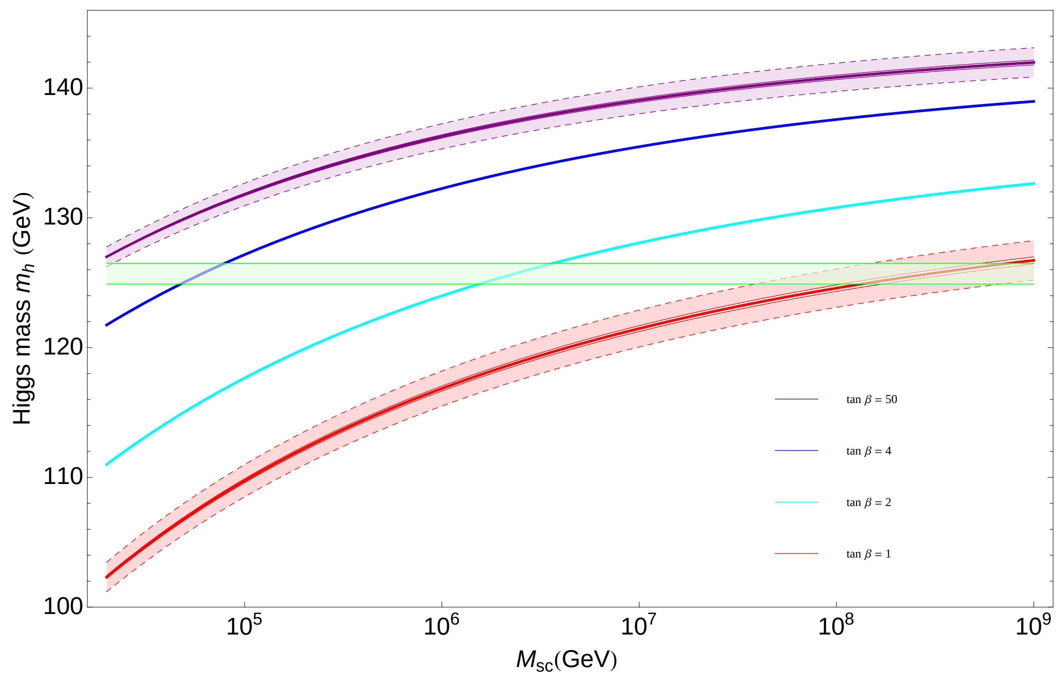


Figure 3.2: The Higgs mass predicted as a function of the scalar masses and $\tan\beta$. The bands at $\tan\beta = 1$ and 50 represent the theoretical uncertainty in the top mass and α_s. The gaugino spectrum is that predicted by the anomaly mediated contribution with the gravitino mass $m_{3/2} = 1000$ TeV, resulting in an approximate mass for the LSP wino of $\sim 2.7 - 3$ TeV (which is roughly the mass necessary for the wino to have the correct cosmological thermal relic abundance to be all of dark matter [103]). The μ term is fixed to be equal to the scalar mass – this threshold has a small but non-negligible effect on the Higgs mass relative to the conventional split supersymmetry spectrum [20, 21]. The A-terms are small.

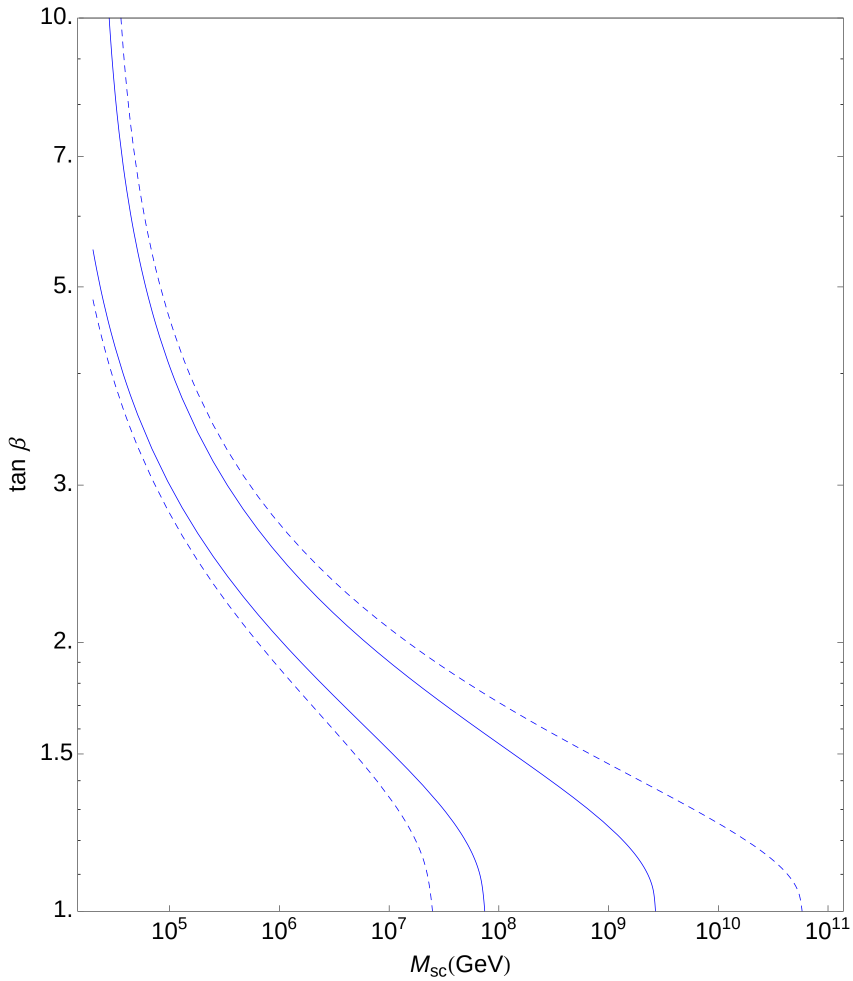


Figure 3.3: The allowed parameter space in the $\tan\beta - M_{sc}$ plane for a Higgs mass of 125.7 ± 0.8 GeV, for $\mu = m_{sc}$. The solid blue lines delimit the 2σ uncertainty. The dashed blue lines show the effect of the 1σ uncertainty in the top mass, $m_t = 173.2 \pm 0.9$ GeV [9]. We take the gaugino spectrum predicted by AMSB (including the heavy Higgsino threshold) with the gravitino mass $m_{3/2} = 500$ TeV, resulting in a wino LSP of 2.6 TeV, and a gluino mass of 14.4 TeV. However, the Higgs mass is highly insensitive to the gaugino spectrum, and a gravitino mass of 50 TeV yields essentially the same plot above.

Higgs mass of 125 GeV. The result, shown in Figures 3.2 and 3.3, is somewhat different from that found in [27]. The Higgsinos are not present in the effective theory beneath m_{sc}, and thus the positive contribution to the running of the Higgs quartic coupling from Higgsino/gaugino loops in the low-energy theory is absent. This means that $m_H \sim 125$ GeV is consistent with heavier scalars than in the split spectrum considered in [27]. In particular, for moderate $\tan\beta$, scalar masses in the $10^3 - 10^4$ TeV range are perfectly allowed, while with light Higgsinos such heavy scalars are only possible if $\tan\beta$ is tuned to be close to 1. Such heavy scalars naturally avoid all flavor problems, giving another impetus to focus on a spectrum with $\mu \sim m_{3/2}$.

Because the Higgsinos have a significant impact on the differential running of gauge couplings, keeping μ heavy noticeably changes the unification prediction. For example, we see in Figure 3.4 that two-loop running predicts a smaller value of the strong coupling constant $\alpha_s(M_z)$ than what is generally found in the 'natural' MSSM. For example, with $\mu = 1000$ TeV, $\alpha_s(M_z) = 0.113$ for gluinos at 1.5 TeV and $\alpha_s(M_z) = 0.111$ for gluinos at 15 TeV (for $\mu = 100$ TeV, $\alpha_s(M_z) = 0.1185$ and 0.117, respectively). Of course this prediction is affected by unknown threshold corrections at the GUT scale, but the values found here are a bit closer to the world average of $\alpha_s(M_z) = 0.1184(7)$ [9], and are very close to more recent determinations using LEP data [104].

Heavy scalars and Higgsinos do moderately better at b-τ unification than the natural MSSM (Figure 3.5), especially at low values of $\tan\beta$. In addition, for small $\tan\beta$,

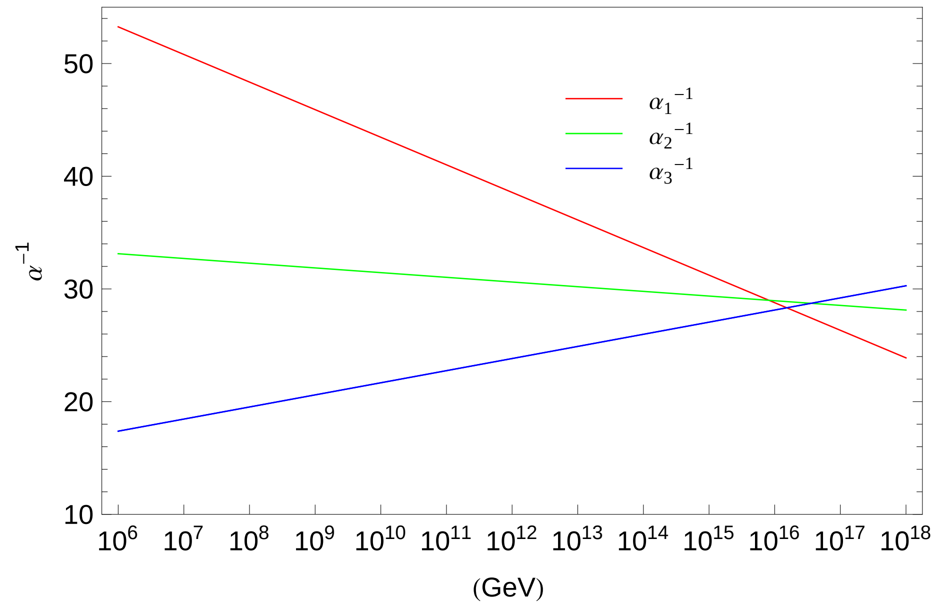


Figure 3.4: The running of the gauge couplings with scalar masses and Higgsinos fixed at 10^3 TeV. Error bands on α_s are at the three-sigma level according to the Particle Data Group [9]. We use $M_{gluino} = 14.4$ TeV, $M_{wino} = 2.6$ TeV, $M_{sc} = \mu = 10^3$ TeV and $\tan\beta = 2.2$ to generate this plot.

the top Yukawa runs relatively strong at the GUT scale, and one would naturally expect significant threshold corrections.

In pure anomaly mediation, the gaugino masses are widely split, with the gluino roughly a factor of ten heavier than then wino. This is due to the same accident as the near cancellation of the one-loop beta function of SU(2) in the MSSM. With a pure GM term (ignoring soft masses), the Higgsino threshold increases the wino and bino masses such that the gluino/wino ratio is reduced to roughly a factor of six. An interesting limit occurs if the Higgses are mildly sequestered from W_{hid} such

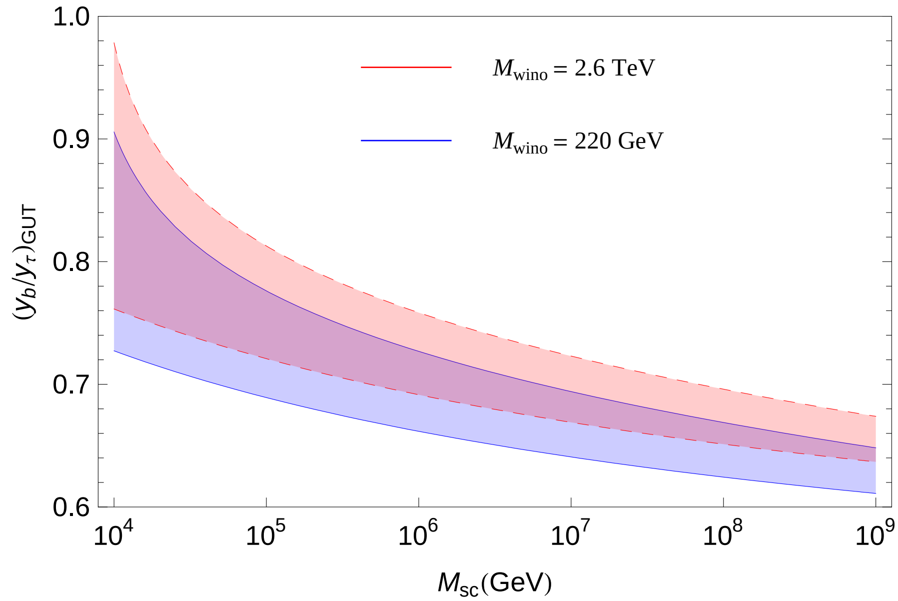


Figure 3.5: The Yukawa coupling ratio (y_b/y_τ) evaluated at the GUT scale as a function of the scalar mass. The gluino masses corresponding to the light and heavy wino masses are 1.5 TeV and 14.4 TeV, respectively. The bands correspond to $\tan\beta = 1.2$ and 50 from top to bottom.

that Planck-suppressed couplings to supersymmetry breaking are absent, but the μ-term comes from $H_u H_d W_0$. In such a limit, the threshold correction suppresses the wino mass, and in fact at leading order in $B\mu/\mu^2$ the wino mass vanishes! Of course, without soft masses, electroweak symmetry breaking at a scale much smaller than $m_{3/2}$ would require $B\mu/\mu^2 \to 1$, in which case the wino retains $\sim 40\%$ of its standard MSSM value. Without sequestering, however, soft masses generally reduce the threshold effect, and the operator $H_u H_d W_{hid}$ adds to the magnitude of the wino mass and thus reduces the large splitting.

3.2.2 New Vector-Like States

As with the μ term, $m_{3/2}$ is a natural mass scale for vector-like states. Additional vector-like states, with big SUSY breaking, can further significantly modify the anomaly-mediated spectrum of gauginos. To preserve gauge coupling unification, we assume that these states are in complete multiplets of $SU(5)$. We have seen in Ch. 2 that in the simple limit that their masses come from a pure GM mass term, they invariably produce a squeezed spectrum among the MSSM gauginos. As defined in Sec. 2.4, the effective number of messengers measures the size of the threshold correction compared to that of one canonical $\mathbf{5}+\bar{\mathbf{5}}$ pair (in standard $SU(5)$ language) with a pure GM mass and no additional scalar soft masses.

A heavy vector-like state whose mass comes only from a superpotential (*i.e.*, supersymmetric) operator would, at leading order in F/M, decouple in such a way as to leave the anomaly-mediated relationships between beta-functions and gaugino masses intact. In the case of a pure GM mass term, the sign of the effective F term for the scalar components of the new states is opposite to that of the superpotential case, and therefore the threshold corrections to gauginos also have sign opposite to the one required to keep the spectrum "anomaly mediated." For one to four sets of vector-like states, this tends to suppress the splitting between the gluino and the wino (or lightest gaugino). For example, with one vector-like state, the one-loop beta-function coefficients above the threshold for $SU(3)$ and $SU(2)$ are $b_3 = (b_3)_{\mathrm{MSSM}} + 1 = -2$ and $b_2 = (b_2)_{\mathrm{MSSM}} + 1 = 2$, respectively. Below the threshold, the coefficients become

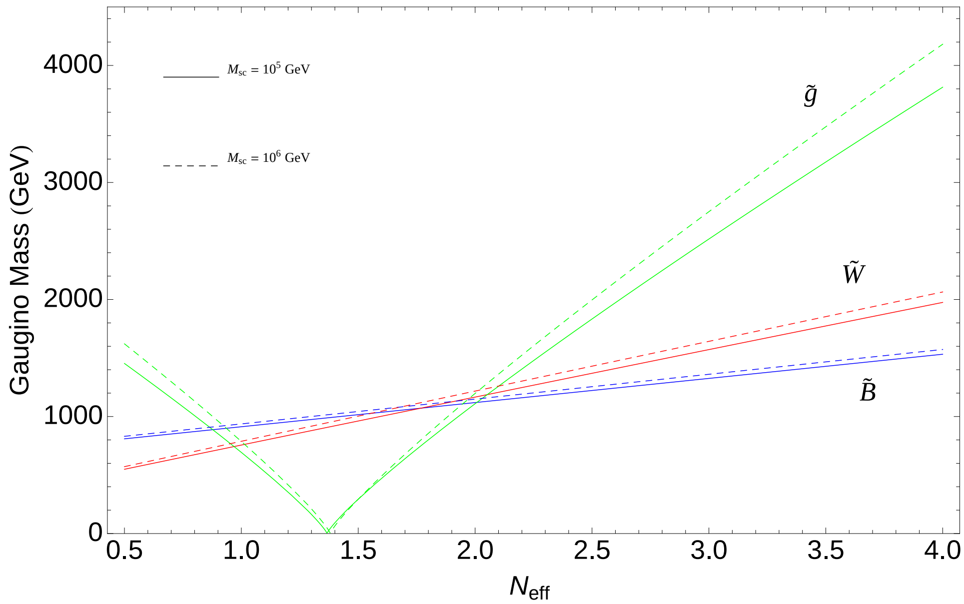


Figure 3.6: The gaugino spectrum as a function of N_{eff} (defined in Sec. 2.4) at two-loop order plus threshold corrections. The other parameters in the model are $m_{3/2} = 70$ TeV, $\tan\beta = 2.2$, and the coefficient for the GM term $\lambda = 1.1$. M_{sc} is again the soft mass for all MSSM scalar superpartners and we set $\mu = M_{sc}$.

$(b_3)_{\rm MSSM} = -3$ and $(b_2)_{\rm MSSM} = 1$, while the gaugino masses (at leading order) are proportional to $(b_3)_{\rm MSSM}+2 = -1$ and $(b_2)_{\rm MSSM}+2 = 3$. Accounting for the hierarchy in gauge couplings, this renders gluinos and winos roughly degenerate. Generally, the gaugino mass coefficients for N messengers will be $(b_i)_{\rm MSSM} + 2N$, where i runs over the gauge groups.

More generally, soft masses for the scalar components of the vector-like states will suppress the threshold correction. In the limit of soft masses much larger than the GM mass, the threshold correction goes to zero, and the resulting spectrum becomes

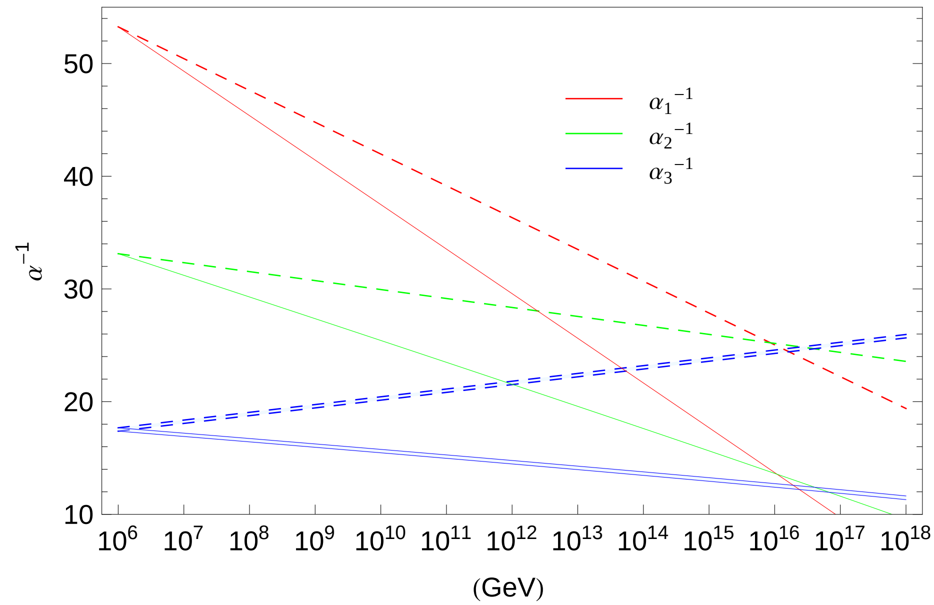


Figure 3.7: The running gauge couplings in the case of $N = 1$ vector-like state (dashed), and $N = 4$ (solid). The scalar masses and Higgsinos are fixed at 10^3 TeV. Error bands on α_s are at the three-sigma level according to the Particle Data Group [9]. We use $M_{gluino} = 14.4$ TeV, $M_{wino} = 2.6$ TeV, $M_{sc} = \mu = 10^3$ TeV and $\tan\beta = 2.2$ to generate this plot.

proportional to $(b_i)_{\text{MSSM}} + N$ – only half the effect. Thus, a more general parameterization of this threshold contribution is $(b_i)_{\text{MSSM}} + 2N_{eff}$, where N_{eff} (defined in Sec. 2.4) is N in the GM limit, and $N/2$ in the limit of large scalar soft masses. In Figure 3.6, we plot the gaugino spectrum as a function of N_{eff}. We see that the ratio of gluino mass to lightest gaugino is always smaller than the pure MSSM case. A squeezed spectrum is of course more hopeful for the discovery of the gluino at a collider for fixed LSP mass.

The vector-like states have a slightly negative effect on unification, as shown in Figure 3.7. For example, if we take the masses of the fermionic components of the messengers to be 10^6 GeV, then for $N = 1$, the central value predicted for $\alpha_s(M_Z)$ is 0.111, while for $N = 4$ it is 0.109. In addition, $b - \tau$ unification is significantly worse. However, if these new states are associated with a model of flavor, Yukawa coupling unification would depend on the full theory.

3.2.3 Dark Matter

One of the compelling motivations for new particles at the weak scale is the idea of WIMP dark matter. In models of the sort we are considering, where R-parity makes the LSP stable, we expect some thermal relic abundance regardless of whether the LSP comprises the majority of the dark matter. And since this is the lightest new particle in the spectrum, it is important to understand what mass it can have.

To begin with, we can consider the conventional anomaly-mediated spectrum, with a wino dark matter candidate [105]. In this case, to achieve the appropriate relic abundance, we require a mass of $\sim 2.7 - 3$ TeV [103]. With conventional anomaly mediation for the gaugino masses, this would make the gluinos inaccessible at the LHC. However, as we have already discussed, with the contributions of the Higgsinos and potentially new vector-like states, the spectrum is naturally squeezed. If it is quite squeezed, it is conceivable that the gluinos will be just at the edge of discovery, even with a thermal relic wino dark matter candidate. Since the direct detection cross section of a pure wino is extremely small [106,107], below $O(10^{-47}\mathrm{cm}^2)$, discovery via direct detection will be extremely difficult.

However, a number of other options are also possible. With a wino LSP, it may simply be that the dark matter is dominantly composed of something else (e.g., axions). In such a case, the LSP can be quite light (from the perspective of cosmological constraints), and almost any spectrum is open to us, including relatively light ($\sim$ TeV) gluinos. Such a wino could be the dark matter if produced non-thermally (e.g., [62, 105, 108], or more recently [109]). Indeed, in the context of minimal Split SUSY models, it is reasonable to expect late-decaying moduli to dilute any thermal LSP abundance, with the dark matter being re-populated by modulus decays. This still favors dark matter lighter than the TeV scale to get the correct relic abundance, and can also pleasingly dilute the troublesome axion abundance down to acceptable levels, for axion decay constants almost as high as the GUT scale [74]. If the bino is the LSP, we must rely upon late-time entropy production to dilute away an otherwise highly overabundant relic.

In each case, there remains the prospect for interesting collider signals. For a thermal relic, we must count on a squeezed spectrum, while non-thermal (or non-WIMP dark matter) cases are generally easier to find. Regardless, the appearance of signals at the LHC will possibly point to a non-standard thermal history.

3.3 New Flavor Physics and Radiative Fermion Masses

In our picture, the supersymmetric flavor problem must be solved in a trivial way, and not with ingenious model-building and gymnastics. Without any special structure to the scalar mass matrices, in particular with no mechanism enforcing scalar mass degeneracy, $K - \bar{K}$ mixing and ϵ_K demand that the first two generations of squarks be as heavy as $\sim$ 1000's of TeV. What about the third generation squarks? They can plausibly be comparable to the first two generations, or at most an order of magnitude lighter.

To give a simple example for theories of flavor leading to the second possibility, consider models where the Yukawa hierarchy is explained by the Frogatt-Nielsen mechanism, with the light generations having different charges under anomalous $U(1)$ symmetries [110, 111]. The anomalous $U(1)$'s are Higgsed by the Green-Schwartz mechanism, and the gauge bosons are lighter than the UV cutoff (string scale), parametrically by a factor of $\sqrt{\alpha}$. Tree-level exchange of this $U(1)$ gauge boson can give SUSY breaking that dominates over Planck-suppressed soft masses. This gives large, different masses to the first two generations, since they are charged under the $U(1)$, but not to the third generation. With an $\mathcal{O}(1)$ splitting between the first two generation scalars, these soft masses must be in the range of at least 1000's of TeV. Planck-suppressed operators will put the third generation scalars in the range of 100's of TeV. Note that we can't imagine the third generation much lighter than a factor of ~ 10 compared to the first two generations. In RG running from high scales, two-loop corrections to the third generation soft masses from the first two generations give large negative soft masses that would lead to color breaking [112].

Thus, if we want to trivially solve the flavor problem, the first two generations of scalars should be in the range of 1000's of TeV, while the stops can be at most an order of magnitude lighter, in the range of hundreds of TeV. Note that the Higgs mass constrains the geometric mean of the left and right stop masses, given by $m_{\tilde{t}} = \sqrt{m_{\tilde{q}_3} m_{\tilde{u}_3}}$, and as we have seen, with $\tan\beta \sim \mathcal{O}(1)$, we can have $m_{\tilde{t}} \sim 10^2 - 10^3$ TeV, so this is perfectly consistent with solving the flavor problem. It is also interesting that the scalars can't be much *heavier* without making the Higgs mass too big. Thus, the absence of flavor violation pushes the scalar masses up, but getting the right Higgs mass doesn't allow these masses to get too large, saturating right around a scale of $\sim$ 1000 TeV. It is of course notable that this is just what is expected from the simplest picture of SUSY breaking we have been discussing.

If we have scalars in just this range, with no special effort to suppress flavor violation in the soft terms, we might be sensitive to new flavor violation in future experiments.

There is another interesting observation about flavor, which provides an additional motivation for a "split" spectrum with gauginos lighter than scalars. Let us suppose that there is indeed large flavor violation in the soft masses. Huge FCNCs are an

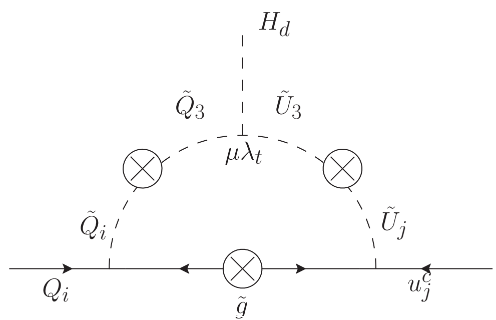


Figure 3.8: Diagram that generates up-type quark Yukawa couplings from the top Yukawa in the case of large mass mixing between flavors, indicated by the crosses on the scalar lines.

obvious worry, but (ignoring detailed issues of the Higgs mass for the moment) one would imagine that these could be decoupled by making the scalars arbitrarily heavy. This is of course correct. But in a theory with no splitting between scalars, Higgsinos, and gauginos, there is a far greater difficulty with flavor that cannot be decoupled by pushing up the scale of SUSY breaking: The large flavor violation, in tandem with a large top Yukawa coupling and the breaking of R-symmetry by the μ term and gaugino masses, radiatively feeds unacceptably large contributions to the up-type quark Yukawa couplings at one-loop. The diagram in Fig. 3.8 yields

$$\delta\lambda_u^{ij} \sim \frac{\alpha_s}{4\pi}\frac{m^2_{\tilde{Q}_{i3}}}{m^2_{sc}}\frac{m^2_{\tilde{U}_{j3}}}{m^2_{sc}}\frac{\lambda_t}{\tan\beta}\frac{\mu\, m_{\tilde{g}}}{m^2_{sc}} \tag{3.5}$$

With $m^2_{ij}/m^2_{sc} \sim O(1)$, tan $\beta \sim O(1)$, and $\mu \sim m_{\tilde{g}} \sim m_{sc}$, this gives a correction to all up-type Yukawa couplings of order $\sim 10^{-2}$, vastly larger than observed.

It is interesting that for our minimally split spectrum, with $\mu \sim m_{sc}$ and $m_{\tilde{g}} \sim 10^{-2}\, m_{sc}$, this correction is roughly of the order of the observed up quark Yukawa coupling. The up quark mass can plausibly arise from this "SUSY slop." Note that the analogous "slop" cannot be significant for the down and electron Yukawa couplings since the corrections are $\propto \lambda_{b,\tau}$ tan β, and for the moderate tan β we are forced to have, the corrections are about 10^{-2} of the observed values.

More generally, supersymmetric theories with a split spectrum allow us to re-open the idea of a radiatively generated hierarchy for Yukawa couplings. The central challenge to building such theories of flavor is the following: the chiral symmetries protecting the generation of Yukawa couplings must obviously be broken, but then what forces the Yukawas to only be generated at higher loop orders [30, 113]? Supersymmetry offers the perfect solution to this problem, since the chiral symmetries can be broken in the Kähler potential, while holomorphy can prevent this breaking from being transmitted to Yukawa couplings in the superpotential. It is then only after SUSY breaking that the chiral symmetry breaking can be transmitted radiatively to generate Yukawa couplings [51]. Unfortunately, it is extremely difficult to realize this idea in a simple way, with a natural supersymmetric spectrum; the degree of flavor violation needed in the soft terms is large, and would lead to huge flavor-changing neutral currents. But in our new picture this is no longer the case: Yukawa couplings are dimensionless and can be generated at any scale, while the FCNCs decouple as the scalars are made heavy.

As we have seen, with the minimal MSSM particle content, only the top Yukawa coupling is large enough to seed the other Yukawa couplings, and thus it is only possible to generate the up quark Yukawa coupling radiatively. Additional vector-like matter near m_{sc} allows the possibility of new large Yukawa couplings and thus more radiatively generated Yukawas. For instance, with a single additional $\mathbf{5}+\bar{\mathbf{5}}$ messenger pair, $(D^c, L) + (D, L^c)$, we can have large mixing Yukawas of the form

$$\lambda_d^i q_i H_d D^c, \lambda_e^i e_i^c H_d L \tag{3.6}$$

as well as large scalar soft mass mixing

$$m_{d_i}^2 \tilde{D}^{c*} \tilde{d^c}^i, m_{l_i}^2 \tilde{L}^* \tilde{l}^i \tag{3.7}$$

between the (D^c, L) and the ordinary d_i^c, l_i. Then, the analogous diagram to Figure 3.8 contributes to down-type quark and charged lepton Yukawa couplings, with $\lambda_t \to \lambda_{d,e}^i$. For $\lambda_{d,e} \sim O(1)$, we can have a radiative origin for the down quark and electron Yukawa couplings.

Given that $m_{\tilde{g}} \sim \frac{\alpha}{4\pi} m_{sc}$, these radiative corrections are parametrically two-loop effects. With additional full vector-like generations, together with heavy-heavy and heavy-light Yukawa couplings to the Higgses, we can also get one-loop corrections through diagrams of the form shown in Figure 3.9. This picture thus easily provides some basic ingredients for the construction of realistic theories where all the fermion masses arise radiatively off the top Yukawa coupling together with other $O(1)$ Yukawa couplings to heavy vector-like states. In Ch. 4, we build a radiative model of flavor

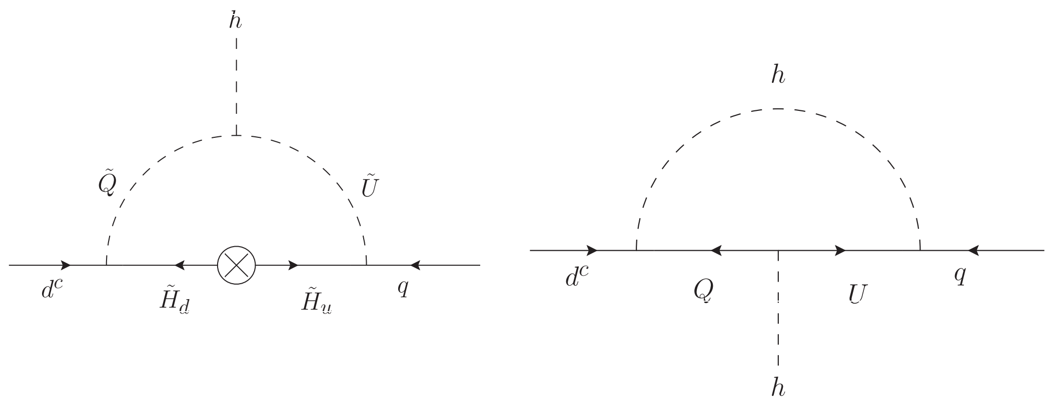


Figure 3.9: Radiatively generated down-type quark Yukawa couplings seeded by heavy messenger-Higgs Yukawa couplings.

along similar lines, with a democratic treatment of all flavors.

3.4 Tests of Un-naturalness

The theoretical developments leading to the development of the Standard Model were greatly aided by concrete experimental evidence for the presence of new physics at short distances not far removed from the scales experimentally accessible at the time. This was most obvious for the weak interactions, which were encoded as dimension six operators suppressed by the Fermi scale. The presence of these operators, together with their $V - A$ structure, were strong clues pointing to the correct electroweak theory. The theoretical triumph of the Standard Model has rewarded us with a renormalizable theory, with no direct evidence at all of higher dimension operators suppressed by nearby scales. Instead of having concrete clues to the structure of new physics through the observation of non-zero coefficients for higher dimension operators–say through a large correction to the S-parameter, a large rate for $\mu \to e\gamma$ or sizable electron EDMs–the main guideline to extending the Standard Model for the past thirty years has been to explain "zero": the *absence* of large quadratically divergent corrections to the Higgs mass, while seeing *no* observable effects in higher dimension operators.

The discovery of a natural supersymmetric theory–as spectacular as it would be–would eventually leave us in a similar position: we would have another renormalizable theory, with no obvious indications for new physics needed until ultra-high energy scales. Amusingly enough, however, the situation is completely different in the un-natural theories we have been discussing in this chapter. The theory has two scales of new physics, $m_{1/2}$ for the gaugino masses and m_{sc} for the scalar masses. As we will see, if we can produce the gauginos, their decays can provide us with unique opportunities to measure or constrain higher dimension operators suppressed by the scale m_{sc}.

Before turning to this discussion, let us first ask an even more basic question: what experimental signals can immediately *falsify* these simply un-natural theories? The existence of any new scalar state beyond the Higgs would immediately exclude simply un-natural models, since it would require an additional tuning, a mechanism to stabilize its mass, or an elaborate family/Higgs symmetry structure. The only important caveat to this is that pseudo-Nambu Goldstone bosons (pNGBs) could still be consistently present. However, if these are light without tuning, they can't be charged under the SM gauge groups, and they can only have higher dimension couplings to SM fields suppressed by their decay constant. We would not expect to produce such states at colliders.

Thus, a second light Higgs, mixing and significantly altering the properties of the Higgs (e.g., [114–116]), would exclude the theories described in this chapter. This makes precision measurements of the Higgs, such as deviations from SM branching ratios, especially important. Large modifications of the Higgs couplings to the W, Z require the existence of a new scalar. Higgs couplings to fermions can be modified by e.g., mixing with new vector-like matter, as can the Higgs width to $\gamma\gamma$. Both ATLAS and CMS reported in their Higgs discovery announcements a ~ 1.7 enhancement of the rate for $h \to \gamma\gamma$ relative to the SM [117–121]. A further CMS analysis based on the full dataset from the first run of the LHC saw this drop to a level that is consistent with the SM to within 1σ [122]; the ATLAS number has though stayed more-or-less constant and disagrees with the SM by about 2σ [123]. Both collaborations however report consistency with the SM in the ZZ and WW decay channels [7, 124, 125]. With only extra fermions, it is a challenge in general to theoretically achieve rates for $h \to \gamma\gamma$ above a factor of 1.5 of the SM value, and even then with some tension relative to precision electroweak observables [126, 127]. For the case of Split SUSY, however, getting the enhancement in the $\gamma\gamma$ channel while leaving WW and ZZ unchanged requires something very specific: new electrically-charged vector-like fermions that are light (about 150 GeV or less) and have electroweak interactions but that do not carry color [128]. Thus, if the hint, albeit quite mild at present, of a deviation in $h \to \gamma\gamma$ persists without an associated discovery of charged particles lighter than ~ 150 GeV, simply un-natural theories will have been conclusively excluded.

In fact, within the framework we are discussing, with only the MSSM field content present, the leading interactions of the Higgs bosons to new, electroweak charged states are suppressed by $1/\mu$, and thus the corrections to Higgs properties are far too small to be seen. Thus, at least this minimal framework could be excluded by any convincing deviation of Higgs properties from SM expectations.

3.4.1 Gaugino Decays and the Next Scale

In a simply unnatural theory, the only new particles that we expect to see at the LHC are the gauginos. The impact of this on the decays of gauginos is profound, for a simple reason: *for $SU(3) \times SU(2) \times U(1)$ fermions that do not carry B or L, there are no renormalizable operators under which they can decay into each other and SM particles.* This simple point was emphasized by [20]. As a consequence, the gaugino decays are necessarily suppressed by a new higher scale.

The scale in question varies depending on the particular process. Gluinos decay

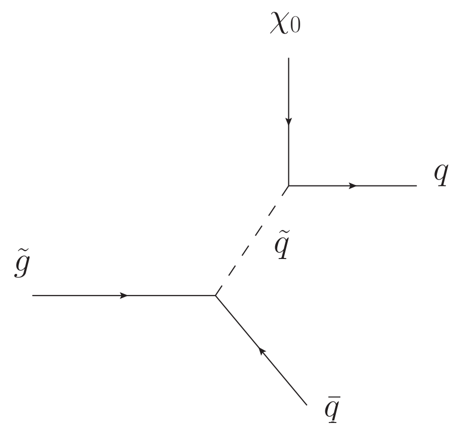


Figure 3.10: Diagram involving a heavy, off-shell squark that yields the dimension-six operator of Eq. (3.8) contributing to gluino decay to the LSP χ^0.

through the diagram in Figure 3.10, which yields the dimension-six operator

$$\frac{g_3 \bar{q}\tilde{g}\bar{\chi}q}{m_{\tilde{q}}^2}. \tag{3.8}$$

The lifetime for such a decay can be quite long, with

$$c\tau \approx 10^{-5}\mathrm{m}\left(\frac{m_{\tilde{q}}}{\mathrm{PeV}}\right)^4\left(\frac{\mathrm{TeV}}{m_{\tilde{g}}}\right)^5 . \tag{3.9}$$

This leads to an interesting immediate observation: the fact that gluinos decay *at all* inside the detector will imply a scalar mass scale within a few orders of magnitude of the gluino mass scale. Moreover, if the gluino decays promptly, without any displacement, we will already know that the scalar mass scale is at an energy scale $\lesssim 100$ TeV, which is at least conceivably accessible to future accelerators.

While this signal places an upper bound on the next mass scale, there are signals that can simultaneously place a quick lower bound. In particular, it is possible to imagine that large flavor violation in the scalar sector could produce clear flavor violation in the gluino decays (e.g., $\tilde{g} \to \bar{t}c$). If so, closing the loop generates sizable flavor-violating four-Fermi operators $\alpha_s^2 q^4/m_{\rm sc}^2$. Even for CP-conserving processes, constraints push this scale to [129] $\sim 10^3\,\text{TeV}$ ($\sim 10^4\,\text{TeV}$ if CP is violated). A combination of a lack of displaced vertices and large flavor violation in gluino decays could quite narrowly place the next scale of physics, without ever having observed a single particle close to the heavy scale.

The quark line above can be closed to yield a chromomagnetic dipole operator as well

$$\frac{g_3^3}{16\pi^2}\frac{m_{\tilde{g}}}{m_{\tilde{q}}^2}\log(m_{\tilde{q}}/m_{\tilde{g}})\tilde{g}_j^i\sigma^{\mu\nu}\tilde{b}G_{i\mu\nu}^j. \tag{3.10}$$

Such an operator will produce dijet + MET (missing transverse energy) signals, but because their rate is suppressed by a loop factor, they should be lost in the overall four jet + MET signals of the off-shell squark decay.

In contrast to gluinos, bino decay proceeds through a dimension-five operator that arises from integrating out the Higgsino, shown on the left in Figure 3.11, namely

$$\frac{g_2 g_1}{\mu}h_i^*\tilde{W}_j^i h^j\tilde{B}\ . \tag{3.11}$$

The suppression by only one power of the heavy scale suggests that these decays will be prompt.

Note that this operator generates a $\tilde{W}-\tilde{B}$ mixing term, which in general will correct the mass of the neutral wino relative to the charged wino by an amount

$\sim m_W^4/(\mu^2 m_{\tilde{W}}) \sim 10^{-7}\text{GeV}(\text{PeV}/\mu)^2(\text{TeV}/m_{\tilde{W}})$. This correction is negligible, even compared to the conventional loop-suppressed mass splitting $m_{\tilde{W}^\pm} - m_{\tilde{W}^0} \approx 150$ MeV [130]. The leading dimension-five operator $\tilde{W}^i_j h^*_i h^l \tilde{W}^j_l$, while correcting the wino mass, does not yield any mass splittings between the usual components. Thus, the mass splitting between charged and neutral winos is a clear test of heavy Higgsinos.

The operator of Eq. (3.11) leads to the decay $\tilde{W}^0 \to h\tilde{B}$. Note, however, that because there is no light *charged* Higgs, there can be no decay $\tilde{W}^\pm \to h^\pm \tilde{B}$ through this operator. Rather, the equivalent decay will arise from the resulting $\tilde{W} - \tilde{B}$ mixing, giving $\tilde{W}^\pm \to W\tilde{B}$. While the $\tilde{W}^\pm \tilde{W}^0$ production cross section is generally smaller than the Higgs production cross section, it is not far off from the associated production cross section (a few hundred fb at $m_{\tilde{W}} \sim 200\,\text{GeV}$ [131]). Consequently, in these models, there are new avenues for Wh production that might be searched for.

Note that this dimension-five operator does *not* give $\tilde{W}^0 \to \tilde{B}Z$. This decay can arise from a dimension-six operator, integrating out the Higgsinos at tree-level (Figure 3.12)

$$\frac{g_1 g_2}{\mu^2} h^\dagger D_\mu h \tilde{W} \bar{\sigma}^\mu \tilde{B}, \tag{3.12}$$

and it can also come from the dimension-five operator obtained by integrating out

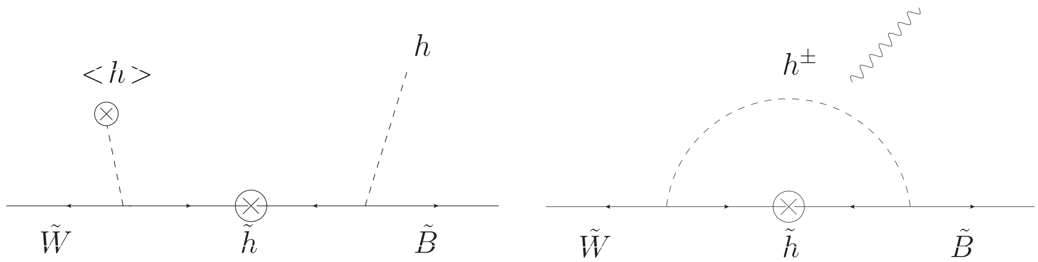


Figure 3.11: Diagrams that contribute to the dimension-five operators of Eq. (3.11) (left) and Eq. (3.13) (right).

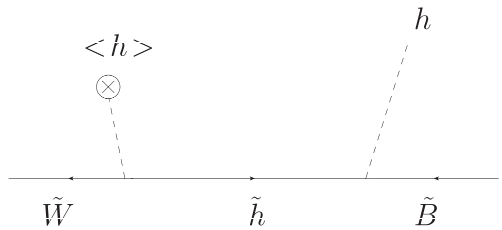


Figure 3.12: Diagram that yields the dimension-six operator of Eq. (3.12).

the Higgsinos at one-loop, generating a dipole operator (cf. right of Figure 3.11)

$$\frac{g_2^2 g_1}{16\pi^2 \mu} \tilde{W}^i_j \sigma^{\mu\nu} \tilde{B} F^j_{i\mu\nu}. \tag{3.13}$$

In either case, with heavy Higgsinos $\tilde{W}^0 \to \tilde{B} Z$ is expected to a rare decay. If the Higgsinos are heavier than ~ 10 TeV, the radiative dimension-five operator dominates the amplitude and we have a branching ratio for this decay $\sim (\alpha/(4\pi))^2 (m_{\tilde{W}}/m_Z)^2$. For heavy enough winos this could be observable. Note that the dimension-six operator can only contribute to $\tilde{W}^0 \to \tilde{B} Z$ but not to $\tilde{W}^0 \to \tilde{B}\gamma$, while the dipole operator gives both. The pure dipole predicts a ratio of the photon to Z final states of just $\sin^2\theta_W / \cos^2\theta_W \sim 1/3$. A measurement of this could establish the dipole operator as the source of the wino decay, and would show that the Higgsinos are heavy enough for the dimension-six operator to be negligible. Alternately, a deviation from this ratio would tell us that the Higgsinos are heavy, but lighter than ~ 10 TeV.

Having enumerated the decay possibilities, we now consider the signatures of gluino production and decay at the LHC. Let us assume a non-squeezed spectrum with $m_{\tilde{g}} > m_{\tilde{W}} > m_{\tilde{B}}$. This offers the possibility of spectacular processes. If the stops are the lightest colored scalars, we have the signal of $t\,\bar{t}\,t\,\bar{t}\,\tilde{B}\tilde{B}$ final states, which yields four tops + MET, where the stops are potentially produced from displaced vertices (if the scalar scale is high enough, or the spectrum is adequately squeezed). More striking is if the decay proceeds as $\tilde{g} \to t\bar{t}\tilde{W}^0$, with $\tilde{W}^0 \to \tilde{B}h$. In such a case we could find final states with 8 b's, four $W^\pm$ and significant MET (and again, possibly displaced vertices). Such a process would have effectively zero background, making the only question for this scenario whether gluinos are produced at all. At 14 TeV and 300 fb^{-1}, we estimate approximately 5 events for ~ 2.5 TeV gluinos (or 3 TeV gluinos for ten times that data). In some cases, the decay $\tilde{g} \to \bar{t}\,b\tilde{W}^+$ will occur, followed by $\tilde{W}^+ \to W^+\tilde{B}$. Note that this final state is very similar (topologically) to the direct decay $\tilde{g} \to \bar{t}t\tilde{B}$.

Let us now consider the possibility that the bottom of the spectrum is reversed and the wino is the LSP. Essentially all the decays should proceed via Higgs emission (if kinematically available), i.e. the decay $\tilde{g} \to \bar{t}t\tilde{B}$ will be followed by $\tilde{B} \to \tilde{W}h$. In contrast, direct decays to charged winos will proceed through $\tilde{g} \to \bar{t}\,b\tilde{W}^-$, with the chargino proceeding to decay into $\tilde{W}^0$, producing a disappearing track.

Thus, to summarize, for the $m_{\tilde{W}} > m_{\tilde{B}}$ case, the final states are $4t+$ MET, $4t\,2h$ + MET, as well as $2t\,2b\,2W+$ MET. For the $m_{\tilde{B}} > m_{\tilde{W}}$ case, the final states are $4t$ + MET, $4t\,2h$ + MET and $2t\,2b$ + MET. It is clear from this list that distinguishing these cases will be non-trivial. However, the W from the chargino decay should be distinguishable from the one that comes from top decay, while direct decay to b's should produce a spectrum of b quarks which are in principle distinct from those from top decay. Finally, the presence of the classic disappearing track signature, once seen, would be a clear sign of the wino LSP.

3.4.2 Gluino Decays and Stop Naturalness

One of the key features of an unnatural theory is that the LR soft masses (the mass mixing between a sfermion charged under $SU(2)$ and its singlet partner of the same flavor) should be negligible. Even with large A and μ, these terms are also proportional to the Higgs vev, and are thus naturally $\sim 10^4$ times smaller than the soft mass-squared terms. This impacts gluino decays in an interesting way.

In more detail, the gluino decay operators are

$$\frac{g_2}{\Lambda_L^2}\tilde{g}\, b_L\bar{t}_L\tilde{W}^- \quad \frac{g_2}{\Lambda_L^2}\tilde{g}\, t_L\bar{t}_L\tilde{W}^0 \quad \frac{g_1}{\Lambda_L^2}\tilde{g}\, t_L\bar{t}_L\tilde{B} \tag{3.14}$$

$$\frac{g_1}{\Lambda_L^2}\tilde{g}\, b_L\bar{b}_L\tilde{B} \quad \frac{g_1}{\Lambda_t^2}\tilde{g}\, t_R\bar{t}_R\tilde{B} \quad \frac{g_1}{\Lambda_b^2}\tilde{g}\, b_R\bar{b}_R\tilde{B}$$

where $\Lambda_L^{-2} = g_3 \sum_i U^L_{i3} \tilde{m}^{-2}_{L,i} U^{L*}_{i3}$, $\Lambda_t^{-2} = g_3 \sum_i U^t_{i3} \tilde{m}^{-2}_{t,i} U^{t*}_{i3}$, and $\Lambda_b^{-2} = g_3 \sum_i U^b_{i3} \tilde{m}^{-2}_{b,i} U^{b*}_{i3}$ are the mass scales obtained from a sum over squark mass eigenstates on the internal line in Figure 3.10, weighted by the matrix U that transforms from the flavor basis to the mass basis.

The key observation here is that we have five distinct decay modes into heavy flavor, $\tilde{g} \to \bar{t}t\tilde{W}^0$, $\tilde{g} \to \bar{b}b\tilde{W}^0$, $\tilde{g} \to \bar{t}b\tilde{W}^+$, $\tilde{g} \to \bar{t}t\tilde{B}$, and $\tilde{g} \to \bar{b}b\tilde{B}$. In contrast, we have only three distinct mass scales in the problem, Λ_L, Λ_t, and Λ_b. Thus, the decay of gluinos into heavy flavor is a highly overconstrained system in the unnatural limit, while for natural theories, cross terms introduce additional parameters into the theory. The heavy flavor branching ratios can easily falsify the unnatural scenario. Alternately, if they are consistent with small A terms, this would place additional fine-tuning strain on the MSSM to accommodate the Higgs mass, though of course, we cannot discount a cancellation that reduces sensitivity to these cross terms.

This discussion does not account for top polarization measurements. Should t_l and t_r be distinguishable, this would introduce yet another quantity into an already overconstrained system. If that, too, could be understood with only the three mass scales, it would provide strong evidence of a simply unnatural theory. Regardless, it would clearly show that scalar masses are not significantly corrected by electroweak symmetry breaking.

3.5 Conclusions

The expectation of a natural resolution to the hierarchy problem has always been the best reason to expect new physics at the TeV scale, accessible to the LHC. Naturalness demands new colored states lighter than a few hundred GeV, needed to stabilize the top loop corrections to the Higgs mass. These colored particles must be accessible at the TeV scale. Dark matter is another reason to expect new particles in the vicinity of the weak scale, but the WIMP "miracle" is not particularly sharp and allows for masses and cross-sections that can vary over several orders of magnitude. Indeed, if we take the simplest picture for dark matter–new electroweak doublets or triplets, annihilating through the W and Z, the needed masses are at 1 or 3 TeV, well out of range of direct production at the LHC. It is only naturalness that forces *colored* particles to be light, with the expectation that they should be copiously produced at the LHC.

On the other hand, naturalness has been under indirect pressure from the earliest days of BSM model-building, and the pressure has been continuously intensifying on a number of fronts in the intervening years. The LHC is now exploring the territory where natural new physics should have shown up. No new physics has yet been seen, and while it is far from the time to abandon the idea of a completely natural theory for electroweak symmetry breaking, the confluence of indirect and direct evidence pointing against naturalness is becoming more compelling. But the Higgs mass $m_H \sim$ 125 GeV, is within a stone's throw of its expected value in supersymmetric theories, and the compelling aspects of low-energy SUSY–precision gauge-coupling unification and dark matter–remain unaltered.

The simplest picture resolving the tensions given by this state of affairs is the minimally split SUSY model we have discussed in this chapter. These models can be easily killed experimentally, for instance, if the hint of the large enhancement to $h \to \gamma\gamma$ without enhancement to $h \to ZZ, WW$ is solidified. As for positive signals, indirect evidence for the heavy scalars can arise since since they are just in the range where they may give rise to interesting levels of FCNCs.

The direct LHC probes of these models walks on a knife's edge of excitement. Obviously if the new fermions are too heavy to be produced we have nothing. But if the gauginos are directly produced, not only do we see new particles, but since their decays can only proceed through higher-dimension operators, we get a number of handles on the presence of a high scale between 10 to 1000 TeV, ranging from displacement or flavor violation in gluino decays, to rare decay modes for the wino/bino. This would be enormously exciting, not only providing dramatic evidence for fine-tuning at the electroweak scale, but giving an indication of new thresholds that are not out of reasonable reach for future accelerators in this century.

As has long been appreciated and repeatedly pointed out, the dark matter motivation does not guarantee that the gauginos will be accessible to the LHC; the LSP could be a 3 TeV wino giving the correct relic abundance. But it is also perfectly possible that they are light enough to be produced. Fortunately, the final states from gluino decays are so spectacular that only a handful need to be produced to confirm discovery. If Nature has indeed chosen the path of un-natural simplicity, we will have to hope that she will be kind enough to let us discover this by giving us a spectrum with electroweak-inos lighter than ~ 300 GeV or gluinos lighter than ~ 3 TeV.

Chapter 4

Split SUSY Radiates Flavor

4.1 Introduction

In this chapter, we build a model where the SM fermion masses are generated radiatively in a Mini-Split setup. The spectrum is outlined in Fig. 4.1: the MSSM scalars as well as all the additional ingredients needed for the model are at the scale m_{sc}, while gauginos are significantly lighter, around 10 TeV.

The flavor model has a $U(1)_F$ symmetry in the UV which forbids the Yukawa couplings. However, unlike previous models, all of the SM matter multiplets are neutral under this symmetry, with only the Higgs fields being charged. Therefore, the UV theory treats all the SM fields democratically, and no special charges are needed for the different generations. SUSY breaking occurs at the scale m_{sc} and seeds spontaneous $U(1)_F$ breaking. This allows tree level Yukawa couplings to be generated

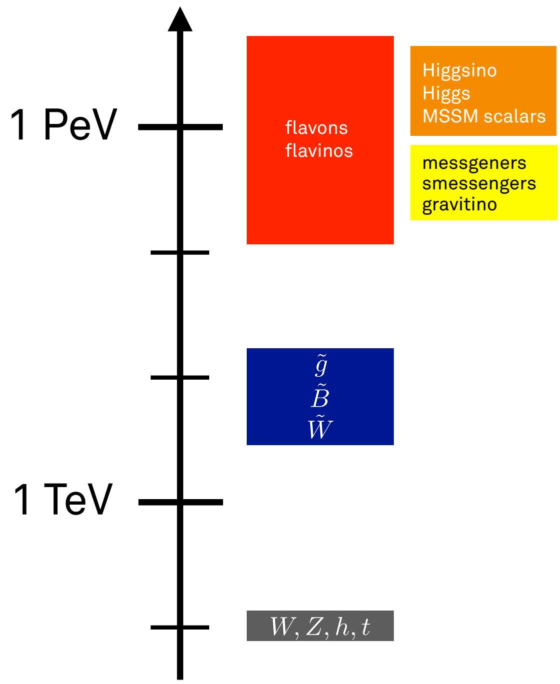


Figure 4.1: The spectrum of the model presented on a log scale. The heaviest known SM particles are at the bottom around 100 GeV. The gauginos are at the 10 TeV scale with the gluino typically heaviest and the Wino typically lightest and closer to 3 TeV. The rest of the spectrum is roughly at the PeV (= 1000 TeV) scale, but they are typically spread out over a couple of decades in mass. As discussed in Sec. 4.3, the messengers mix with the squarks and sleptons.

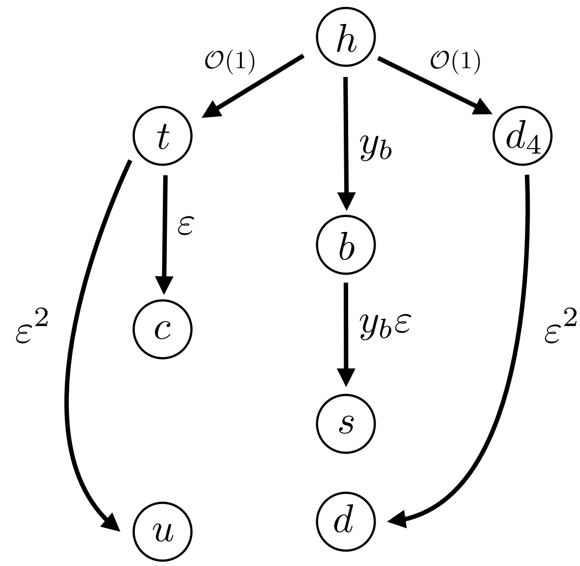


Figure 4.2: A schematic representation of the model given in this work. The top and d_4 fields have $\mathcal{O}(1)$ couplings to the Higgs, while the coupling of the b is somewhat smaller. The 2$^{\text{nd}}$ generation gets one-loop couplings from the 3$^{\text{rd}}$ generation with ε being a loop factor. The top and d_4 seed Yukawa couplings for the up and down which are parametrically two-loop size.

for the 3$^{\text{rd}}$ generation fermions. The relative smallness of the bottom and tau Yukawa couplings to the top Yukawa comes from a modest and technically natural tuning, but this is the only hierarchy not automatically explained by this model. Radiative corrections from the $U(1)_F$ breaking sector generate one-loop Yukawa couplings for the 2$^{\text{nd}}$ generation. Finally, the 1$^{\text{st}}$ generation Yukawas are generated by two-loop diagrams of sfermions which have large flavor breaking in their SUSY-breaking masses. A schematic representation of the fermion mass hierarchies is given in Fig. 4.2. The CKM matrix also has the right structure, with the small parameter required for a small bottom Yukawa being the reason that the Cabibbo angle is larger than a loop factor. Additionally, this model preserves the predictions of gauge coupling unification and dark matter of Mini-Split SUSY.

The organization of this chapter is as follows. In Sec. 4.2 we describe our model and give the parametric sizes of elements of the Yukawa matrices and translate those into the SM fermion masses and mixing angles. In Sec. 4.3, we calculate the predictions of the model in detail including the spectrum of fields at m_{sc} as well as the SM fermion masses and mixings. We also present a benchmark point in parameter space which faithfully reproduces SM flavor observables (*cf.* Figs. 4.11 and 4.12). In Sec. 4.4 we describe the constraints on the model and potential future phenomenology, and we conclude in Sec. 4.5.

4.2 A Model of Flavor

In this section we give a schematic description of the model and describe the parametric sizes of the SM flavor parameters. We show the full spectrum in Fig. 4.1, present a benchmark in Sec. 4.3, and the details of the calculations in the Appendices. We begin by describing the dynamics needed for the up sector, and we will cover the rest of the SM fermions in subsequent sections.

Table 4.1: Charge assignments of the Higgs and up-sector messenger fields. Here R_p denotes the usual R-parity. Note that the MSSM fields q and u fields are neutral under $U(1)_F$.

Field	U(1)$_F$	$SU(3) \times SU(2) \times U(1)$	R_p
H_u, H_d	∓ 2	$(\mathbf{1}, \mathbf{2})_{1/2} + (\mathbf{1}, \mathbf{2})_{-1/2}$	$+$
Q, $\bar{Q}$	± 1	$(\mathbf{3}, \mathbf{2})_{1/6} + (\bar{\mathbf{3}}, \mathbf{2})_{-1/6}$	$-$
U, $\bar{U}$	± 1	$(\bar{\mathbf{3}}, \mathbf{1})_{-2/3} + (\mathbf{3}, \mathbf{1})_{2/3}$	$-$

4.2.1 Up Sector

Our model is an extension of the MSSM with the spectrum broadly described in Sec. 3.2. The basic premise is that the hierarchy of masses between generations is a hierarchy in the number of loops. Crucial to the setup is a means to forbid tree-level Higgs Yukawas to all but the 3$^{\text{rd}}$ generation. Satisfying this criteria, we must then ensure the remaining chiral symmetries are broken in stages to parametrically separate the first two generations. Furthermore, the different couplings of the generations occur solely as a consequence of linear algebra. We make no *ad hoc* or symmetry-based distinctions between them. To prevent Yukawa couplings at tree level, we add a new $U(1)_F$ gauge group under which the Higgs superfields are charged, but all other MSSM fields are neutral. We discuss complications associated with a new gauge group such as anomalies in App. A. For the up sector, we also introduce one additional

Table 4.2: The set of flavon fields needed to break $U(1)_F$, along with their charge assignments.

Field	U$(1)_F$	$SU(3) \times SU(2) \times U(1)$	R_p
$\phi_{1,2},\ \bar{\phi}_{1,2}$	± 1	$(\mathbf{1}, \mathbf{1})_0$	$+$
$\chi_{1,2},\ \bar{\chi}_{1,2}$	∓ 3	$(\mathbf{1}, \mathbf{1})_0$	$+$
$\xi,\ \bar{\xi}$	∓ 2	$(\mathbf{1}, \mathbf{1})_0$	$+$

generation of vector-like messenger quarks, Q, $\bar{Q}$ and U, $\bar{U}$ which have $U(1)_F$ charges such that a primordial Yukawa coupling $\lambda_U QUH_u$ can be written down. The set of fields needed for the up sector as well as their charges are given in Table 4.1.

In order to generate any Yukawa couplings, we need to spontaneously break $U(1)_F$. This requires the introduction of "flavon" fields shown in Table 4.2. As described in App. B, we need each of these fields in order to get a potential that generates the flavon VEVs required for SM Yukawas. The flavons get soft masses from the same mechanisms as the MSSM matter. Taking ϕ as an example, the soft terms are given by

$$V_\phi^{\text{soft}} = \frac{1}{2}(m_\phi^2)_{ij}\, \phi_i^\dagger \phi_j + \frac{1}{2}(m_{\bar{\phi}}^2)_{ij}\, \bar{\phi}_i^\dagger \bar{\phi}_j - (b_{ij}^\phi\, \phi_i \bar{\phi}_j \,+\, \text{h.c})\,. \tag{4.1}$$

Once we include the D-terms arising from $U(1)_F$, the flavon scalar potential is analogous to the Higgs potential in the MSSM, so there is a large region of parameter space that can be chosen such that all the ϕ fields acquire VEVs. Since all the dimensional parameters in the potential are of the same order, we naturally get $\langle \phi_i \rangle \sim m_{\rm sc}$. The potential minimization is described in more detail in App. B.

From the field content of Tables 4.1 and 4.2, we can write down a general superpotential

$$
\begin{aligned}
W_{\rm up} &= \lambda_U Q\, U\, H_u + \bar{\lambda}_U \bar{Q}\, \bar{U}\, H_d + f^q_{ij}\, q_i\, \bar{Q}\, \phi_j + f^u_{ij}\, u_i\, \bar{U}\, \phi_j \\
&\quad + \mu_Q\, Q\bar{Q} + \mu_U\, U\bar{U} + \mu\, H_u H_d + \mu^\phi_{ij} \phi_i \bar{\phi}_j \,, \qquad (4.2)
\end{aligned}
$$

where we have ignored the interactions of the χ and ξ flavons for now. The f couplings have flavor indices, but the λ couplings to the Higgs are just numbers. The μ-terms are all of order $m_{\rm sc}$ and are generated via the Giudice-Masiero mechanism (see Sec. 2.3 and Sec. 3.2 for more detail), so all the states described in Tables 4.1 and 4.2 will have mass $\mathcal{O}(m_{\rm sc})$.

4.2.1.1 Top Yukawa

With these ingredients, we can generate a top Yukawa coupling at tree level, with all other Yukawas still zero. This arises from the messenger exchange diagram in Fig. 4.3. When $U(1)_F$ is broken by the VEV of ϕ, the f couplings in Eq. (4.2) generate a mixing between the MSSM-like fields and the heavy vector-like fields. The f couplings have an index in ϕ doublet space as well as an index in flavor space. We can choose our ϕ basis such that only ϕ_1 gets a VEV and $\langle \phi_2 \rangle = 0$. If we set ϕ to its VEV and ignore interactions of the propagating ϕ for now, we see that f^q and f^u are just column vectors, so they are both rank 1. We can thus choose bases for q_i and

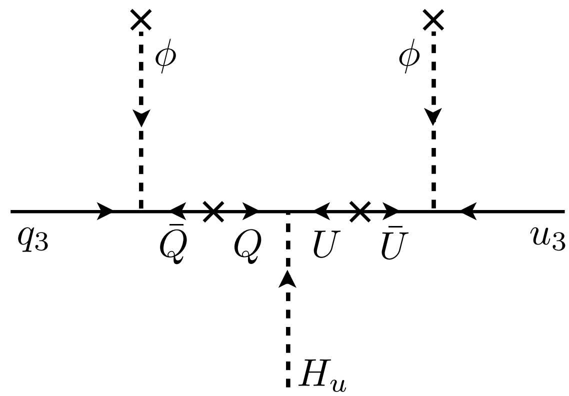


Figure 4.3: Feynman diagram for generating the top Yukawa coupling.

u_i such f^q_{ij} and f^u_{ij} are only non-zero in the "3" direction in flavor space. This basis now defines the top quark. It is the only up-type quark to mix with the vector-like quarks. Thus, Fig. 4.3 only generates a top Yukawa coupling. This mechanism, which is similar to that of previous works such as [34–36, 44, 132], allows a UV theory where all the SM quarks are treated democratically to generate only a top Yukawa coupling at tree level.

To calculate the top Yukawa from the interactions given in Eq. (4.2), we need to rotate the fields as described above. We can make the schematic argument of the previous paragraph more rigorous as follows: without loss of generality, we can use the $U(3)_q$, $U(3)_u$ symmetries that exist in the limit of zero f^q, f^u couplings to remove any interaction between the 1$^{\text{st}}$ generation quarks and the flavons. In a generic basis, both ϕ_1 and ϕ_2 get VEVs and we use the residual $U(2)$ symmetries to decouple the 2$^{\text{nd}}$ generation q and u fields from them. For example, from the original q_2 and q_3, we get

$$q_3' = \frac{f_{22}^q\,\phi_2\,q_2 + (f_{31}^q\,\phi_1 + f_{32}^q\,\phi_2)q_3}{\sqrt{(f_{22}^q\,\phi_2)^2 + (f_{31}^q\,\phi_1 + f_{32}^q\,\phi_2)^2}}, \tag{4.3}$$

and q_2' is the orthogonal linear combination. Here and throughout we will use the name of a field to represent a VEV when the context is clear. We can now define the following two matrices R and F

$$q_i = R_{ij} q_j' \qquad F_{ij}^q = R_{ik}^\dagger f_{kj}^q, \tag{4.4}$$

where R is the matrix that rotates between the interaction and mass eigenbases for the quarks, and F is the rotation of the f couplings into the mass basis. We make an analogous rotation for the u fields and their couplings. By construction, only q_3' couples to the flavon VEVs. Since this defines the 3$^{\text{rd}}$ generation, we drop the $'$ notation for this post-rotation state hereafter.

Having performed the appropriate rotations on the quarks and f couplings, we are at last in position to calculate the contribution to the top Yukawa from Fig. 4.3, getting

$$y_t = \frac{\lambda_U\, F_{3i}^q F_{3j}^u\, \phi_i \phi_j}{\mu_Q \mu_U}. \tag{4.5}$$

We see that for dimensionless factors of $\mathcal{O}(1)$ and all dimensionful factors of the same order, $\sim m_{\text{sc}}$, we get an $\mathcal{O}(1)$ top Yukawa. Of course, if there are no hierarchies in the parameters in Eq. (4.5), then calculating y_t requires us to go beyond the double-VEV insertion approximation of Fig. 4.3. Rather, after $U(1)_F$ symmetry breaking, we need to fully diagonalize the $q_3 - Q$ and $u_3 - U$ mass eigenstates. We save the details of

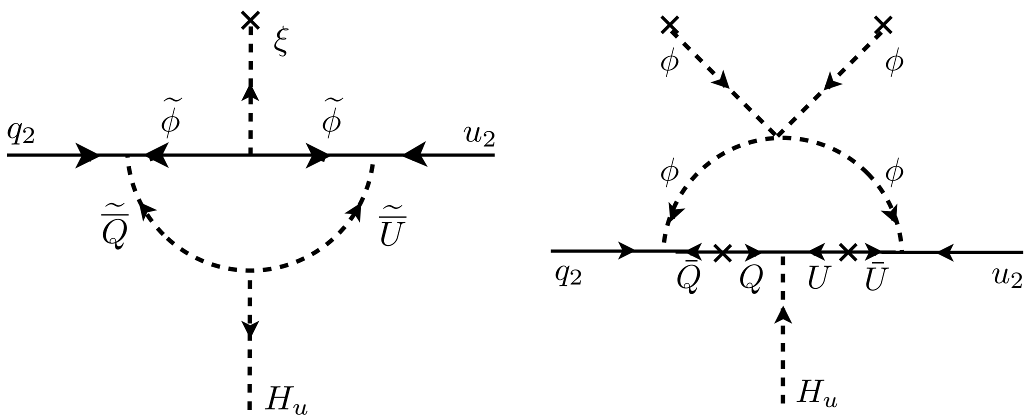


Figure 4.4: Feynman diagrams for generating the charm Yukawa coupling. We use the convention that fields which get VEVs such as ϕ and H_u have tildes over their fermions, while fields which do not get VEVs such as Q and u have tildes over their scalar components.

this discussion for Sec. 4.3.3, but we stress that a full treatment of the top Yukawa maintains its $\mathcal{O}(1)$ parametric size.

4.2.1.2 Charm Yukawa

The $U(1)_F$-breaking dynamics which generate a tree-level top Yukawa coupling also generate a charm Yukawa at one loop. This occurs through the two processes shown in Fig. 4.4. These two diagrams contain the same superpotential f^q and f^u couplings from Eq. (4.2), but we must perform a SUSY rotation to get from the flavon-messenger diagram to the flavino-smessenger diagram. While the $\langle\phi\rangle$ can be rotated so it only points in one direction, there are still two propagating fields, so we can define the 2nd generation of quarks as the linear combination that does not couple to $\langle\phi\rangle$ but does couple to the propagating ϕ. This defines the 1st generation as the quark which does not couple to ϕ at all.

Because of the $U(1)_F$ gauge symmetry, there is a D-term of the form

$$V^D = \frac{g_F^2}{2}\left(\phi_i^\dagger\phi_i - \bar{\phi}_i^\dagger\bar{\phi}_i + ...\right)^2 . \tag{4.6}$$

This generates a flavon four-point coupling, allowing us to draw the diagram on the right side of Fig. 4.4. This diagram must connect a ϕ that does not get a VEV to one that does in order to generate a charm mass. This can only happen if there is misalignment between the basis where the VEV points in a single direction and the basis where the mass matrix is diagonal. While this generically occurs for our flavon potential, the size of the flavon-messenger diagram is suppressed by this misalignment.

Thus, we need to construct the supersymmetrized version of this diagram, a flavino-smessenger diagram, to get a charm Yukawa of the right size. Clearly this is only possible if the flavino $\tilde{\phi}$ has a Majorana mass. As we can see from Table 4.2, the following superpotential operators are allowed

$$W = \lambda_{ij}\phi_i\phi_j\xi + \bar{\lambda}_{ij}\bar{\phi}_i\bar{\phi}_j\bar{\xi} + \lambda'_{ij}\phi_i\chi_j\bar{\xi} + \bar{\lambda}'_{ij}\bar{\phi}_i\bar{\chi}_j\xi . \tag{4.7}$$

These generate the desired flavino mass if ξ gets a VEV. This is the mechanism shown on the left side of Fig. 4.4, which turns out to be the dominant contribution to the charm mass and justifies the inclusion of the ξ flavon in the theory. As we will show in App. B, the ξ and $\bar{\xi}$ flavons serve several other important functions, which explains the flavon content of Table 4.2.

The diagrams in Fig. 4.4 also generate Yukawa couplings of the form $q_3 u_2$ and $q_2 u_3$ which are parametrically one loop. They also give small corrections to the top Yukawa coupling. We will give a detailed description of the computation of the one-loop Yukawa couplings in App. C.1, with the dominant contribution to charm given in Eq. (C.1).

4.2.1.3 Up Yukawa

Finally, we can generate an up quark Yukawa coupling and fill out the rest of the Yukawa matrix through the diagram in Fig. 4.5. In Sec. 3.3 we remarked that these diagrams have the correct parametric size to generate the up quark mass, and we utilize this here. This diagram is one loop, but it has a chirality flip coming from the gluino mass rather than a primordial Yukawa coupling used in the processes of Figs. 4.3 and 4.4. Therefore, this diagram will be suppressed by $m_{\tilde{g}}/m_{\rm sc}$, which in the Mini-Split framework is just a loop factor (see Sec. 3.2). Therefore, the up Yukawa coupling generated by the diagram in Fig. 4.5 is parametrically of two-loop size.

The coupling to the Higgs still comes from the top Yukawa coupling, but here we use the fact that the squark soft masses are anarchic in flavor space as the source of flavor breaking. In the mass insertion approximation [133], one can imagine $\tilde{q}_3$ and $\tilde{u}_3$ coupling to the Higgs, and then each being converted to a different flavor by an $\mathcal{O}(1)$ mass insertion. Fig. 4.5 is drawn in this way, but for truly anarchic mixing, a

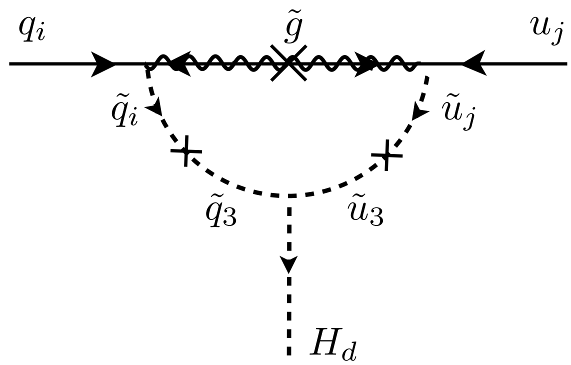


Figure 4.5: Feynman diagram for generating the up Yukawa coupling.

better picture is that the squarks that couple to the Higgs have couplings between the gluino and all three flavors of quarks. Here we see that it is crucial that the mass of the squarks be much above the weak scale, because if not, the mass insertion would be constrained to be small, and the loop diagram in Fig. 4.5 would be too small to generate the up mass. The expression for the up Yukawa and related mixing diagrams is given in Eq. (C.19), where flavon VEV insertions are summed to all orders to diagonalize the squark-smessenger masses.

Thus we see that with the fields introduced in Tables 4.1 and 4.2, we can get an up-type Yukawa matrix which is parametrically of the form

$$\mathbf{y^u} \sim \begin{pmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix}, \tag{4.8}$$

where ε is a loop factor. This matrix gives quark masses $(m_t, m_c, m_u) \sim v(1, \epsilon, \epsilon^2)$, which is the right power counting to match the measured quark masses. The structure of the model is shown diagrammatically in Fig. 4.2. In Secs. 4.3.3, 4.3.4, and App. C we will give more explicit computations of the quark Yukawa couplings and show how the SM can be numerically reproduced.

4.2.2 Down and Lepton Sectors

Because unification is a feature of SUSY even in the split regime, we build a model that is manifestly consistent with SU(5) unification.[1] Therefore, we must add a vector-like E messenger field which has the same SM quantum numbers as the MSSM right handed electron, and the same $U(1)_F$ charge as Q and U to complete the $\mathbf{10}$ representation. In order to generate down and lepton type Yukawa couplings, we must also add a vector-like $\bar{\mathbf{5}}$ representation. Thus we have a full vector-like generation of messengers charged under $U(1)_F$. The additional particle content needed to generate the down and lepton Yukawa couplings is given in Table 4.3, while the full particle content of our model is given in Table A.1 in the Appendix.

The up-type field content can be described in SU(5) language as $\mathbf{10}_i\mathbf{10}_j\mathbf{5}_H$ where i and j are SM flavor indices. Similarly, both the down and lepton type Yukawas can be described as $\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H$. Therefore, in the rest of this section we describe the generation of down-type Yukawa couplings, but the leptons can be derived by trivial replacements within $SU(5)$ representations.

As described in Sec. 3.2, the Mini-Split SUSY scenario works for $\tan\beta$ of moderate

[1]We do not attempt to solve the doublet-triplet splitting problem for the Higgs that is ubiquitous in all SUSY GUT constructions.

Table 4.3: Fields needed to generate the down and lepton Yukawa couplings in addition to those in Tables 4.1 and 4.2, as well as their charges.

Field	$\mathrm{U}(1)_F$	$SU(3) \times SU(2) \times U(1)$	R_p
E, $\bar{E}$	± 1	$(\mathbf{1}, \mathbf{1})_1 + (\mathbf{1}, \mathbf{1})_{-1}$	$-$
D, $\bar{D}$	∓ 3	$(\bar{\mathbf{3}}, \mathbf{1})_{1/3} + (\mathbf{3}, \mathbf{1})_{-1/3}$	$-$
L, $\bar{L}$	∓ 3	$(\mathbf{1}, \mathbf{2})_{-1/2} + (\mathbf{1}, \mathbf{2})_{1/2}$	$-$
ℓ_4, $\bar{\ell}$	0	$(\mathbf{1}, \mathbf{2})_{-1/2} + (\mathbf{1}, \mathbf{2})_{1/2}$	$-$
d_4, $\bar{d}$	0	$(\bar{\mathbf{3}}, \mathbf{1})_{1/3} + (\mathbf{3}, \mathbf{1})_{-1/3}$	$-$

size, so the bottom and τ Yukawa couplings are parametrically smaller than that of the top quark. Therefore, if we were to use the same dynamics as we used for the up-type quarks, we would expect, for example, $m_d/m_u \sim m_b/m_t$. Because it is of critical importance that the down quark be comparable in mass or heavier than the up quark, we enhance the structure of the model to fix this relation. We add an additional vector-like down-type quark pair: d_4 and its conjugate partner $\bar{d}$, which are neutral under $U(1)_F$. Unlike the D, d_4 can mix with the SM d_i because they have the same (trivial) $U(1)_F$ charge, and we have an additional "barred" version of the flavon coupling, as $\bar{d}$ couples to $\bar{\chi}$.

With this field content, we can write the following superpotential

$$\begin{aligned} W_{\text{down}} &= \lambda_D \, Q \, D \, H_d + \bar{\lambda}_D \, \bar{Q} \, \bar{D} \, H_u + f^d_{ij} \, d_i \, \bar{D} \, \chi_j + \bar{f}_i \, \bar{d} \, D \, \bar{\chi}_i \\ &\quad + \mu_D \, D\bar{D} + \mu^d_i \, d_i\bar{d} + \mu^\chi_{ij} \, \chi_i\bar{\chi}_j \, , \end{aligned} \tag{4.9}$$

where again all the dimensionless couplings are $\mathcal{O}(1)$ and all the dimensionful terms are $\mathcal{O}(m_{\rm sc})$. We can now choose a basis in flavor space such that μ^d only points in one direction, and this direction picks out the fourth generation of d. This shows that the fourth generation d_4 and $\bar{d}$ will be heavy while the remaining three generations will be massless before electroweak symmetry breaking.

After choosing μ^d to point only in the '4' direction, there is still a residual $U(3)_d$ flavor symmetry in the absence of the f^d coupling. This symmetry exists even if f^d has an $\mathcal{O}(1)$ entry in the '4' direction in d flavor space. Thus, it is technically natural for all the f^d couplings to the SM-like d triplet to be small. This is the scenario we take in this model, namely

$$f^d \sim \begin{pmatrix} y_b & y_b \\ y_b & y_b \\ y_b & y_b \\ 1 & 1 \end{pmatrix} , \tag{4.10}$$

in a generic basis where only μ^d has been rotated into the fourth component. We have dropped $\mathcal{O}(1)$ coefficients in each entry. Here y_b is parametrically the size of the bottom (or τ) Yukawa coupling, and the choice of the coupling of the form of

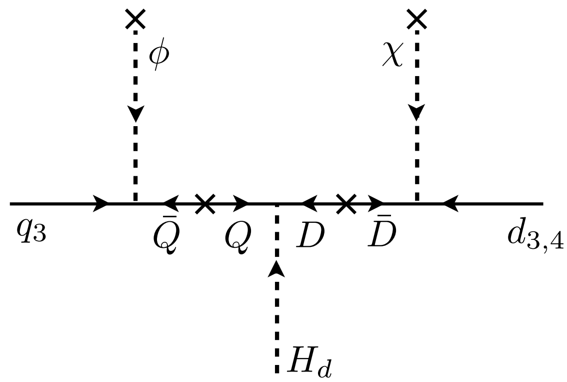


Figure 4.6: Feynman diagram for generating the bottom and d_4 Yukawa couplings.

f_d represents a technically natural tuning of order 10%. This is the only parametric hierarchy in the flavor sector not explained by our model.

We now see that there is a process analogous to that of Fig. 4.3 for the bottom and d_4 quarks shown in Fig. 4.6. As in the top case, we can pick a basis where the χ VEV is only in one direction, and then we can use the $U(3)_d$ symmetry to make the f^d coupling to the χ VEV parametrically $f^d_{\langle\chi\rangle} \sim (0, 0, y_b, 1)$. Since we have a fourth generation, the down Yukawa matrix at the scale of $U(1)_F$ breaking is now 3×4 and it is given by the outer product of $f^d_{\langle\chi\rangle}$ with the corresponding coupling from the q doublet $f^q_{\langle\phi\rangle} \sim (0, 0, 1)$ computed in the previous section.

One-loop 2$^{\text{nd}}$ generation masses proceed in nearly the same fashion as in the up sector through the diagrams in Fig. 4.4 with up-type quarks replaced by down-type, and χ replacing ϕ where necessary. The flavino-smessenger diagram requires the use of the $\chi\phi\bar{\xi}$ coupling given in Eq. (4.7). This shows that the $U(1)_F$ charge assignments given in Table 4.2 are optimal for this model because they allow the generation of both up- and down-type flavino diagrams. The one-loop strange mass diagrams require d_2 to couple to χ, so they are parametrically of size $y_b\,\varepsilon$, where ε is again the loop factor. This is because the parameterization of Eq. (4.10) is natural only if all SM-like couplings to χ are $\mathcal{O}(y_b)$, thus the one-loop diagram has a small coupling. The parametrics of this model then predict that $m_s/m_b \sim m_c/m_t$, a relation that is good to within a factor of a few in nature.

Finally, we can fill out the rest of the Yukawa matrix with the process analogous to that shown in Fig. 4.5. Besides the obvious substitution of u with d, the main difference is that the coupling to the Higgs now comes from the fourth generation down squark instead of the sbottom. That Yukawa coupling is $\mathcal{O}(1)$ instead of $\mathcal{O}(y_b)$. Because d_4 has the same quantum numbers as the SM d_i, we expect that SUSY breaking soft terms mix $\tilde{d}_4$ strongly with all the SM down-type squarks. Therefore the fourth generation Yukawa coupling can be transmitted to all the other down-type squarks parametrically at two-loop order. Thus, we see that adding this fourth generation changes the incorrect relation of $m_d/m_b \sim m_u/m_t$, to the much more accurate one $m_d/m_t \sim m_u/m_t$ because the fourth generation Yukawa and that of the top Yukawa are the same parametric size.

Putting all the results together, the Yukawa matrix in the down sector is parametrically of the form

$$\mathbf{y^d} \sim \begin{pmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & y_b\,\varepsilon & y_b\,\varepsilon \\ \varepsilon^2 & y_b\,\varepsilon & y_b \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix} . \tag{4.11}$$

Here, y_b is the approximate bottom Yukawa coupling, which is somewhat larger than $\varepsilon \simeq g^2/16\pi^2$, the loop factor. While they are not so different in size, we keep track of the parametrics separately so the different physical mechanisms can be more easily understood. This Yukawa matrix is 4×3 because it describes the coupling of 4 d's to 3 q's. This matrix holds at the scale $m_{\rm sc}$ where $U(1)_F$ is broken. At lower scales, the d_4 can be integrated out because it has a large supersymmetric mass, and the fourth row of the matrix can simply be truncated at leading order.

After this truncation, we have a 3×3 matrix which gives the quark masses as $(m_b, m_s, m_d) \sim v(y_b, y_b\,\varepsilon, \varepsilon^2)$, and we have $\varepsilon < y_b < 1$. This shows that the down sector has a parametrically different hierarchy than the up sector. Instead of equal steps going down in generation, this model explains why the ratio of the strange to bottom mass is smaller than down to strange. The full cascading structure of the quark masses in this model is shown in Fig. 4.2.

As explained above, the structure of the leptons is nearly identical with q replaced by e and d replaced by ℓ. The most important change is that the diagram analogous to Fig. 4.5 for the leptons has a bino exchange instead of a gluino. Thus we get that

$m_e/m_d \sim g_1^4/g_3^4 \simeq 0.03$, where two of the factors of the gauge coupling come from the coupling to the gaugino, and two more come from the AMSB gaugino soft mass formula. Here we have run the gauge couplings up to $m_{\rm sc} \simeq 1000$ TeV where this diagram is generated. The parametric estimate for the relative size of the electron and down is somewhat small, but it is not too far off. We now see that our model successfully predicts the masses of the SM fermions at the parametric level, and all that remains is the mixing angles between the quarks.

4.2.3 CKM Matrix

At scales well below $m_{\rm sc}$, we have the following parametric Yukawa matrices taken from Eqs. (4.8) and (4.11):

$$\mathbf{y^u} \sim \begin{pmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix}, \qquad \mathbf{y^d} \sim \begin{pmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & y_b\,\varepsilon & y_b\,\varepsilon \\ \varepsilon^2 & y_b\,\varepsilon & y_b \end{pmatrix}. \tag{4.12}$$

In order to compute the CKM matrix, we use the standard procedure of finding the matrices which diagonalize $\mathbf{y^u}$ and $\mathbf{y^d}$. In particular, we have

$$V_u^\dagger\,\mathbf{y^{u\dagger}}\mathbf{y^u}\,V_u = 1/v^2\,\mathrm{diag}(m_u^2, m_c^2, m_t^2)\,, \tag{4.13}$$

where V_u acts on the "q" indices of $\mathbf{y^u}$. There is an analogous formula for $\mathbf{y^d}$. From Eq. (4.12) we can compute

$$V_u \sim \begin{pmatrix} 1 & \varepsilon & \varepsilon^2 \\ \varepsilon & 1 & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix}, \qquad V_d \sim \begin{pmatrix} 1 & \varepsilon/y_b & \varepsilon^2/y_b \\ \varepsilon/y_b & 1 & \varepsilon \\ \varepsilon^2/y_b & \varepsilon & 1 \end{pmatrix}, \tag{4.14}$$

where we have taken $\varepsilon \ll y_b \ll 1$. In reality, we will soon see that $\varepsilon/y_b \simeq \sin\theta_c \simeq 0.2$ where θ_c is the Cabibbo angle and not that much smaller than 1.

To compute the CKM matrix, we simply take

$$V_{\text{CKM}} = V_u^\dagger V_d \simeq V_d, \tag{4.15}$$

where the second relation comes from the fact that V_u more closely approximates the unit matrix than does V_d. This parametric relation, predicts, for example, $|V_{us}||V_{cb}| \simeq |V_{td}|$, which holds very well in nature.

The above discussion is only applicable to the absolute value of the elements of the rotation matrices, but in general, we expect every element of $\mathbf{y^u}$ and $\mathbf{y^d}$ to have independent phases. Taking $(\mathbf{y^u})_{33}$ from Eq. (4.5) as an example, we see that all the λ, F and μ couplings will have phases, so the total coupling will also have a phase. Similar arguments can be made about the other elements of the Yukawa matrices, with different couplings entering the computations so they will have independent phases. Therefore, in the absence of cancellation, the physical phase of the CKM matrix will also be $\mathcal{O}(1)$. In Sec. 4.3.4 we will describe a point in the parameter space of this model which reproduces the Standard Model more accurately, but, just from the parametric estimates of this section, we see that we have succeeded in explaining nearly all the hierarchies of the SM flavor sector.

4.3 Computing the Spectrum

In this section we give the details of the computation of the masses of the various states in the theory, including the gauginos, the light Higgs, and of course the SM fermions. We also describe a benchmark point in parameter space so that we can give definite numbers for every effect for at least one point in parameter space. The details of the benchmark including the reproduction of the SM flavor parameters is described Sec. 4.3.4.

4.3.1 Gaugino Spectrum, Unification, and Dark Matter

In our framework, the gauginos are the only states that are relatively light and could be probed in the near future, so it is important to have a precise understanding of the mass hierarchy for phenomenological reasons. As stated in Sec. 3.2, the gaugino masses are on the anomaly-mediated trajectory above the messenger scale $\mu_M \sim m_{\rm sc}$. Because SUSY is broken at the messenger scale, integrating out the messengers will induce threshold corrections that will deflect them from their anomaly-mediated values. The Higgs states will also shift the gaugino masses, but they must be treated with care because one of the states remains light.

Our flavor model requires one set of $\mathbf{10} + \overline{\mathbf{10}}$, containing Q and U and their conjugates, and two sets of $\mathbf{5} + \bar{\mathbf{5}}$, one containing D and the other d_4. The soft masses and b-terms for the messengers are generated by the Giudice-Masiero (GM) mechanism [12], as in Eq. (2.23). The b-term generated by the GM operator is opposite in sign to that obtained from a superpotential mass term which explains why the messengers do not decouple. As described in Sec. 2.4.1, the threshold correction due to each messenger pair depends on the supersymmetric messenger mass μ_M, the holomorphic SUSY breaking mass, b_M, and the soft mass m_M^2. We define the following dimensionless ratios for a given messenger pair M:

$$r_M = |b_M|/|\mu_M|^2 \qquad c_M^2 = m_M^2/|\mu_M|^2\,. \tag{4.16}$$

We can compute the threshold correction for a given messenger pair with Dynkin index C_M defined as $1/2$ for a fundamental of $SU(N)$ and Y^2 for hypercharge. The threshold correction is then given by

$$\Delta m_{\tilde{i}} = -2e^{i\theta_M} C_M \frac{\alpha_i}{2\pi} f_t(y_1,y_2)\frac{|b_M|}{|\mu_M|}, \qquad f_t(y_1,y_2) = \frac{y_1 \log y_1 - y_2 \log y_2 - y_1 y_2 \log(y_1/y_2)}{(y_1-1)(y_2-1)(y_2-y_1)}, \tag{4.17}$$

with $y_i = M_i^2/|\mu_M|^2$, where $M_{1,2}^2$ are the eigenvalues of the scalar messenger mass-squared matrix with $M_1 > M_2$ and are given by

$$y_1 = 1 + c_M^2 + r_M, \qquad y_2 = 1 + c_M^2 - r_M. \tag{4.18}$$

The phase is defined as $\theta_M = \arg(b_M/\mu_M)$ and vanishes if the contact terms in Eq. (2.23) are absent, which is the pure GM limit, since in that case both b_M and μ_M arise from the same operator. In general, the phase will be non-zero, and we work in a convention where μ_M is real.

For the Higgs doublets, we are taking[2] $\mu_H \sim m_{\rm sc}$, so they act as an additional messenger pair that contributes its own threshold correction. Because one linear combination of the doublets is tuned to be light, the form of the threshold correction is different:

$$\Delta m_{\tilde{i}} = -\frac{\alpha_i}{4\pi}|\mu_H|e^{i\theta_H}\sin 2\beta\frac{m_A^2}{|\mu_H|^2 - m_A^2}\log\frac{|\mu_H|^2}{m_A^2}, \qquad m_A^2 = \frac{2r_H}{\sin 2\beta}|\mu_H|^2, \tag{4.19}$$

where $r_H = b_H/|\mu_H|^2$ and m_A is the physical pseudoscalar mass which is approximately degenerate with the rest of the heavy Higgses. Note that with our conventions, there is an overall sign here relative to expressions found elsewhere in the literature [134]. Here we work in the convention where b_H is real, so $\theta_H = \arg(\mu_H)$. Since $\tan\beta = \mathcal{O}(1)$ and $\mu_H \sim m_A = \mathcal{O}(m_{sc})$, the Higgsino threshold corrections are comparable in size to those of the messengers. Furthermore, as emphasized in [134], the phase freedom allows for a rich spectrum of gaugino masses, since interference between the various contributions can lead to wino, bino, or gluino LSP.

We now describe the parameters of the gaugino sector for our benchmark point. The spectrum contains a 3.0 TeV wino LSP for suitable dark matter phenomenology, which we will discuss below. For consistency with the SM flavor analysis, we integrate

[2]Here we make it clear that μ_H and b_H are the parameters in the Higgs potential, but, in this context, the Higgs multiplet is another messenger.

out all heavy states at 1000 TeV. The threshold corrections can then be calculated as described above, using the Dynkin indices in App. D. We then run down all the masses to the TeV scale and include any appropriate pole mass corrections. Since we have not considered the lepton sector in any detail, we simply assume the parameters are the same as those for the quarks in the same GUT multiplet. For simplicity, we have taken all of the phases in the messenger sector to be π (except for d_4, l_4), and take $\theta_H = 0$, which means the Higgsino threshold correction is opposite in sign to the contribution from AMSB. Generalizing to $\mathcal{O}(1)$ phases does not change the picture significantly. To obtain our benchmark spectrum with a wino LSP and a decently-sized gluino mass (needed for 1$^{\text{st}}$ generation Yukawas), we take $m_{3/2} = 1100$ TeV. Table 4.4 contains all the messenger and Higgs sector input parameters relevant for calculating the gaugino spectrum in the way described above.

The discussion above was predicated on the assumption of no mixing between quark and messenger fields. However, as described in Sec. 4.3.3.1, once the flavons get VEVs there is mixing between squarks and smessengers as well as mixing between 3$^{\text{rd}}$ generation quarks and messengers. In computing the gaugino spectrum, we take this mixing into account, and the detailed formula is given in App. D. In fact, proper accounting of mixing decreases the messenger threshold corrections by a factor of a few, since these are dominated by Q and we have large q_3-Q mixing. We find that the messenger corrections are about an order of magnitude smaller than the AMSB soft masses. The gaugino pole masses are $m_{\tilde{W}} = 3.0$ TeV, $m_{\tilde{B}} = 13.3$ TeV, and $m_{\tilde{g}} = 20.9$

Table 4.4: Benchmark parameters for the messenger and Higgs sectors. c_H is fixed by the requirement of a light Higgs state. The c column for D contains two values because here we take different soft masses for D and $\bar{D}$; similarly for d_4.

Messenger	μ_M	c_M	r_M	θ_M
Q	1000	1.17	1.1	π
U, E	1000	1.58	1.15	π
D, L	750	3.0, 3.46	2.0	π
d_4, l_4	728	3.36, 3.81	0.5	0
H	2400	fixed	7.8	0

TeV.

Taking these gaugino masses, we can examine gauge coupling unification. The Mini-Split framework differs from regular Split SUSY only in that μ_H is large, i.e. at the same scale as the sfermions. An analysis was carried out in Sec. 3.2 with a similar gaugino spectrum and $\mu_H = m_{sc} = 1000$ TeV, where it was shown that raising μ_H results in good unification with no messengers, with a predicted $\alpha_3(m_Z) = 0.111$ as compared to the experimental value of $\alpha_3(m_Z) = 0.118 \pm 0.003$ [135]. This is consistent with unification because there are in general unknown threshold corrections at the GUT scale of $\mathcal{O}(1/4\pi)$.[3] Therefore, we see that with our field content, the model

[3] In [136] it was argued that this spectrum is inconsistent with unification, but that work requires that the gauge couplings unite much more precisely than the parametric size of the threshold

is consistent with unification. The cases of $N = 1$ and $N = 4$ messengers were also studied in Ch. 3, with sfermions, messengers, Higgsinos and heavy Higgses all introduced into the two-loop running at a common scale of m_{sc}. Unification still works well and occurs at a slightly larger scale, with a larger coupling at unification and a slightly smaller predicted $\alpha_3(m_Z)$, as N is increased. Our extra matter charged under the SM, i.e. the messenger sector and fourth down-type generation, corresponds to $N = 5$. The gauge couplings do not blow up because the messengers are heavy. In fact, because of the high messenger scale, perturbative control is retained even for $N = 6$. For our benchmark with $N = 5$, the unification scale is 1.1×10^{16} GeV, $\alpha^{-1} = 9.3$ at unification, and $g_3 - g_2 = 0.05$ at the GUT scale corresponding to a predicted $\alpha_3(m_Z) = 0.109$.

Finally, we can summarize the dark matter scenario, which is qualitatively very similar to that described in Sec. 3.2. Because the μ-term for the Higgs is so much larger than the gaugino masses, the wino and bino do not mix with one another or with the Higgsino and are very nearly pure states. If the wino is the LSP, then it has a weak scale annihilation cross section and will behave as a usual WIMP. It will have the right relic abundance if it has a mass around 3 TeV [103]. In this case, there would be WIMP annihilations in regions of high dark matter density such as the galactic center, and these could be looked for as indirect dark matter detection signals. Results from various telescopes [137–139] have placed stringent constraints

corrections.

on thermal wino dark matter which are in tension with this scenario for the standard dark matter halo profiles. On the other hand, for profiles that are less steep or cored near the galactic center, this scenario is still viable.

One could imagine many other dark matter scenarios consistent with the Mini-Split framework and the model presented here. For example, if the wino is lighter than 3 TeV, then it will only make up some of the dark matter, but the rest could be made up of another particle such as an axion. Alternatively, if the LSP is produced non-thermally [62, 105, 109], then its mass could be heavier than 3 TeV. Our model could also produce a bino LSP with different choices of the parameters in Table 4.4. While a thermal bino would overclose the universe, it could be nonthermal, or its abundance could be reduced by the co-annihilation mechanism [140]. From this analysis we see that while the flavor dynamics do not directly affect the dark matter story, the two sectors fit well together in the framework of Mini-Split Supersymmetry.

4.3.2 Higgs Mass and Quartic

Since SUSY is broken well above the scale of electroweak symmetry breaking, $SU(2)$ is preserved to a very good approximation, and thus, as in previous Split SUSY models, the tuning in the Higgs sector produces one light doublet, which includes the SM-like Higgs, and one heavy doublet, with degenerate scalars of mass m_A. The leading contributions to the mass of the light Higgs are the usual ones in Split SUSY models. At tree level, there is a contribution from the D-term of the SM $SU(2)\times U(1)$

gauge group, and there are loop contributions arising from the large splitting between the top and stop quarks. These are analyzed in detail for a 125 GeV Higgs in Sec. 3.2 and [27]. If the scalars all have a common mass m_{sc}, then the Higgs mass essentially depends only on m_{sc} and $\tan\beta$ (aside from a very slight dependence on the wino and gluino masses). A 125.7 GeV Higgs mass [141, 142] implies $\lambda = 0.26$ at the weak scale. Running this up to a scale of $m_{sc} = 1000$ TeV and taking only the gauginos to be below m_{sc} gives $\lambda = 0.058$. Both the tree level and one loop contributions to the quartic depend on $\tan\beta$, and a quartic of the right size can be obtained in the MSSM with $m_{sc} = 1000$ TeV if $\tan\beta = 2.2$.

In this model, there are additional subdominant contributions to the Higgs quartic, so the relationship between the Higgs mass and $\tan\beta$ will be modified. The first of these arises from "non-decoupling D-terms" [143, 144] from the new $U(1)_F$. The D-term is of the form

$$V^D = \frac{g_F^2}{2}\left(\phi_i^\dagger\phi_i - \bar{\phi}_i^\dagger\bar{\phi}_i - 3\chi_i^\dagger\chi_i + 3\bar{\chi}_i^\dagger\bar{\chi}_i - 4\xi^\dagger\xi + 4\bar{\xi}^\dagger\bar{\xi} - 2H_u^\dagger H_u + 2H_d^\dagger H_d + ...\right)^2 , \tag{4.20}$$

where the ellipses include terms with the messenger fields which do not get VEVs. Expanding this out generates a Higgs quartic. We can also integrate out the flavons and use the fact that they get VEVs to generate additional Higgs quartics. These contributions are shown in Fig. 4.7. Thus, we generate the coupling

$$\frac{\lambda_F'}{2}\left(H_u^\dagger H_u - H_d^\dagger H_d\right)^2 \to \frac{\lambda_F}{2}\left(H^\dagger H\right)^2 \tag{4.21}$$

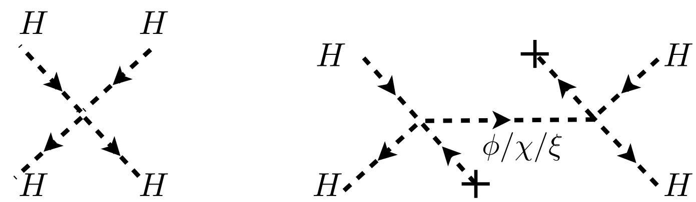


Figure 4.7: Feynman diagrams showing the Higgs quartic generated by the $U(1)_F$ D-term.

$$\lambda_F = 4g_F^2 \cos^2 2\beta \left(1 - \frac{1}{2}g_F^2 \sum_{\phi,\chi,\xi} (qv)_i (m^2)^{-1}_{ij} (qv)_j\right), \tag{4.22}$$

where the sum is over real and imaginary components of all flavon species. Here, (qv) is a vector of the flavon VEVs multiplied by their $U(1)_F$ charges, where the charge is the same for both real and imaginary components. The matrix $(m^2)^{-1}$ is the inverse of the flavon mass squared matrix in the vacuum, and we take all the flavon VEVs to be well above the electroweak scale. In the limit where $U(1)_F$ is Higgsed supersymmetricly, λ_F must go to zero, which will occur as a perfect cancellation between the two terms in Eq. (4.22). In a general region of parameter space where the soft masses and the supersymmetric mass are comparable, there is still a partial cancellation between the two terms in Eq. (4.22), with λ_F about an order of magnitude smaller than $4g_F^2$. For the benchmark described in this section, λ_F, which comes from the $U(1)_F$ D-term, is 0.013, compared to the tree-level MSSM value of 0.037.

The new vector-like states in this model have large couplings to the Higgs, so they will contribute to the Higgs quartic through loops. As these loops must vanish in the supersymmetric limit, they are sensitive to the splitting between scalar and fermion masses. Therefore, the effects of the stops are parametrically larger than those of the new vector-like states. We here compute the full one-loop contribution to the Higgs quartic in the unbroken electroweak theory. Because the (s)tops mix with messengers, it is difficult to disentangle the different effects, and we compute all the one-loop threshold corrections in the mass basis. The diagrams are scalar bubble, triangle, and box diagrams, as well as fermion box diagrams and external line corrections from Higgs wavefunction renormalization. Since our Higgsino mass is $\mathcal{O}(m_{\rm sc})$, we also consider the MSSM contributions from mixed gaugino-Higgsino boxes and contributions to the Higgs field-strength renormalization. In the benchmark, the up-type new generation contributes 0.014 while the new down-type fields contribute $9{\times}10^{-4}$ to the Higgs quartic. In a realistic model, there would also be contributions from the lepton sector, which we estimate to be $1/N_c$ of the down contribution. Once we sum up all the tree level and one-loop contributions, we obtain the right Higgs quartic and mass with $\tan\beta = 1.8$. Therefore, we see that while the effects from the model are indeed subdominant, they need to be taken into account to properly compute the spectrum.

4.3.3 Mass Eigenstates and Wavefunction Renormalization

Before computing the SM flavor parameters in detail, it is necessary to address effects that can induce $\mathcal{O}(1)$ changes to the basic arguments of Sec. 4.2. They are the full diagonalization of the (s)quark-(s)messenger fields after $U(1)_F$ breaking and the one-loop wavefunction renormalization. We stress that the parametric hierarchies given by loop counting are left intact by these considerations, but they can have important numerical effects. We consider them in turn.

4.3.3.1 Diagonalization

Once the flavons get VEVs, the UV distinction between quark and messenger superfields breaks down. In the fermion sector, only the 3^{rd} generation mixes at tree level. For q_3 and u_3, we need only consider the 2×2 mixing with the Q, U messengers. We denote the mass eigenstates as q_3' and Q' with the lower case q' representing SM states, while the capital Q' is a state with mass $\sqrt{\mu_Q^2 + |F^q_{3i}\,\phi_i|^2}$; μ_Q is the supersymmetric mass for the messengers defined in Eq. (4.2), and F is a rotation of the superpotential coupling defined in Eq. (4.4). We here take the convention where μ_Q is real. The mixing is then parameterized as

$$\begin{pmatrix} q_3 \\ Q \end{pmatrix} = \begin{pmatrix} c_q & s_q^* \\ -s_q & c_q \end{pmatrix} \begin{pmatrix} q_3' \\ Q' \end{pmatrix}, \tag{4.23}$$

with

$$c_q = \frac{\mu_Q}{\sqrt{\mu_Q^2 + |F_{3i}^q\,\phi_i|^2}}, \qquad s_q = \frac{F_{3i}^q\,\phi_i}{\sqrt{\mu_Q^2 + |F_{3i}^q\,\phi_i|^2}}, \tag{4.24}$$

and analogous expressions for u and d. For notational compactness, in this section we often use the same notation for both the scalar field and its VEV. In the case that $F_{3i}^q\,\phi_i$ is real, c_q, s_q just become cosine and sine of a rotation angle. After rotating to mass eigenstate basis, Eq. (4.5) for the top Yukawa is modified to

$$y_t = \lambda_U s_q s_u, \tag{4.25}$$

where we recover our earlier formula in the limit $\mu \gg F\,\phi$.

Since d_3, d_4, and $\bar{d}$ couple to flavon VEVs, the diagonalization in the down sector is more complicated. The fermion mass matrix takes the form

$$(\bar{d}\ \bar{D}) \begin{pmatrix} 0 & \mu_d & \bar{f}_i\bar{\chi}_i \\ F_{3i}^d\chi_i & F_{4i}^d\chi_i & \mu_D \end{pmatrix} \begin{pmatrix} d_3 \\ d_4 \\ D \end{pmatrix}. \tag{4.26}$$

To find the SM down quark eigenstate d_3', we solve for the null space of the matrix above, yielding

$$d_3' = \frac{\left(\mu_d\mu_D - F_{4i}^d\chi_i\bar{f}_j\bar{\chi}_j\right)^* d_3 + \left(F_{3i}^d\chi_i\bar{f}_j\bar{\chi}_j\right)^* d_4 - \left(\mu_d F_{3i}^d\chi_i\right)^* D}{\sqrt{|\mu_d\mu_D - F_{4i}^d\chi_i\bar{f}_j\bar{\chi}_j|^2 + |F_{3i}^d\chi_i\bar{f}_j\bar{\chi}_j|^2 + |\mu_d F_{3i}^d\chi_i|^2}}, \tag{4.27}$$

where $\bar{f}$ is defined in Eq. (4.9), and F^d is analogous to F^q, derived from f^d in Eq. (4.9). Our expression for the bottom Yukawa is thus replaced by

$$y_b = \lambda_D\,s_q \frac{\mu_d F_{3i}^d\chi_i}{\sqrt{|\mu_d\mu_D - F_{4i}^d\chi_i\bar{f}_j\bar{\chi}_j|^2 + |F_{3i}^d\chi_i\bar{f}_j\bar{\chi}_j|^2 + |\mu_d F_{3i}^d\chi_i|^2}}. \tag{4.28}$$

Once we perform this rotation, d'_3 decouples, and we are left with a 2×2 Dirac mass matrix that we diagonalize in the usual way. We summarize the product of rotations as

$$\begin{aligned} d^i_{\text{mass}} &= \Gamma^{\dagger ij}_{d}\, d^j_{\text{gauge}}, \\ \bar{d}^i_{\text{mass}} &= \Gamma^{\dagger ij}_{\bar{d}}\, \bar{d}^j_{\text{gauge}}, \end{aligned} \tag{4.29}$$

where $d \equiv (d^3,\, d^4,\, D)$ contains both quark and messenger fields and $\bar{d} \equiv (\bar{d},\, \bar{D})$.

For scalars, due to the anarchic mixing among the squarks from their soft masses, diagonalization is more involved, resulting in 5×5 matrices for $\tilde{q}$, $\tilde{u}$ and 7×7 for $\tilde{d}$. Just as with the fermions, the 3[rd] generation mixes directly with the messengers via the ϕ or χ VEVs. Additionally, the 2[nd] generation also has a tree-level coupling to the messengers through the $\bar{\phi}$ or $\bar{\chi}$ VEVs.[4] For example, in the q sector, we have

$$\mathcal{L} \supset \tilde{\bar{Q}}\, \tilde{q}_i\, F^q_{ij}\, \mu^{\phi^*}_{jk}\, \langle \bar{\phi}^*_k \rangle \, + \, \text{h.c.}\,. \tag{4.30}$$

We do not attempt an analytic diagonalization of the scalar sector, but we perform rotations numerically for the analysis of our benchmark that recovers the Standard Model. For future reference, our convention for rotation matrices is the following (*e.g.* for the q sector):

$$\tilde{q}^i_{\text{mass}} \, = \, \Gamma^{\dagger ij}_{\tilde{q}}\, \tilde{q}^j_{\text{gauge}}, \tag{4.31}$$

[4]This provides an interesting example of the importance of supersymmetry to our model. Without the holomorphicity and non-renormalization properties of supersymmetric theories, we would expect to generate tree-level Yukawas for the 2[nd] generation from the VEV, $\langle \bar{\phi}^* \rangle$, and we would need special potentials in the flavon sector that only broke $U(1)_F$ symmetry with unbarred fields. Supersymmetry allows us to take more generic flavon potentials, while forbidding the barred-flavon VEV Yukawa coupling to SM fermions.

where $\tilde{q} \equiv (\tilde{q}^1, \tilde{q}^2, \tilde{q}^3, \tilde{Q}, -\epsilon\tilde{\bar{Q}}^*)$ contains both squark and smessenger fields. There is an analogous expression for the down sector that involves $\tilde{d} \equiv (\tilde{d}^1, \tilde{d}^2, \tilde{d}^3, \tilde{d}^4, \tilde{D}, \tilde{\bar{d}}^*, \tilde{\bar{D}}^*)$.

The flavon sector contains its own nontrivial rotations after $U(1)_F$ breaking. The flavino matrix is 11×11 and the flavon matrix 20×20, since CP is generically broken and the real and imaginary scalar components mix. For the former, the $U(1)_F$ gaugino, $\tilde{Z}'$, mixes strongly with the flavinos, and thus after symmetry breaking we simply count it among their number. Let $\tilde{\Phi}_{\text{gauge}} \equiv (\phi, \chi, \xi, \bar{\phi}, \bar{\chi}, \bar{\xi}, \tilde{Z}')$ be the fermion components of the superfields appearing in Eqs. (4.2), (4.7), and (4.9) plus the gaugino, with Φ_{gauge} the corresponding scalars, arranged with the ten real-component fields followed by the ten imaginary ones. Then, in analogy with Eq. (4.31), we write

$$\begin{aligned} \Phi^i_{\text{mass}} &= \Gamma^{\dagger ij}_{\Phi}\, \Phi^j_{\text{gauge}}, \\ \tilde{\Phi}^i_{\text{mass}} &= \Gamma^{\dagger ij}_{\tilde{\Phi}}\, \tilde{\Phi}^j_{\text{gauge}}. \end{aligned} \tag{4.32}$$

The one additional subtlety in the flavon sector is that one must identify the zero-mass eigenstate that corresponds to the longitudinal mode of the heavy $U(1)_F$ boson. We work in a unitary gauge where this state never appears in calculations with flavons in mass eigenstate basis.

In addition to the one-loop contributions to the Yukawa couplings discussed in Sec. 4.2, there are additional contributions from loops of Higgsinos and electroweak gauginos shown in Fig. 4.8. Unlike the gaugino contribution to the 1$^{\text{st}}$ generation mass from Fig. 4.5, there is no gaugino mass insertion in this diagram and thus no parametric suppression. Therefore, one would expect that these diagrams are

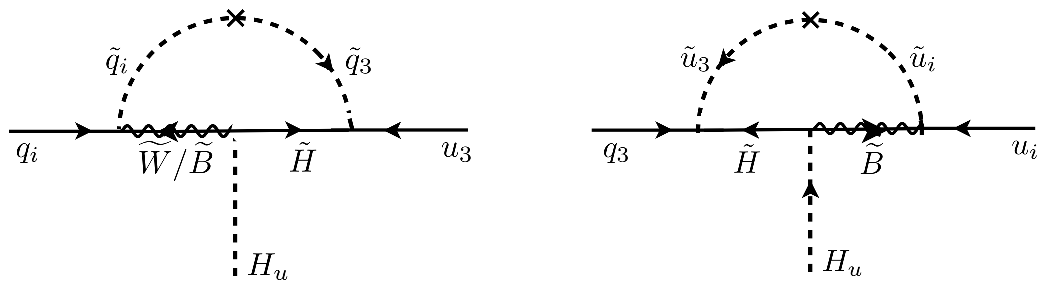


Figure 4.8: One-loop electroweak contribution to off-diagonal Yukawa couplings. These are potentially important for '13' and '31' entries of the Yukawa matrix.

important, but they turn out to be small. One needs the full treatment of rotation matrices above to understand why they are suppressed. Taking the full fermion and scalar rotations, we get the following contribution to the up-type Yukawa matrix:

$$y^u_{3i} = \lambda_U \, s_u \, r^q_i \, \Gamma^{ij^*}_{\tilde{q}} \, \Gamma^{4j}_{\tilde{q}} \, C_{Fk} \frac{\alpha_k}{\pi} \, \mathcal{G}(\mu_H, \, m_{\mathrm{ino_k}}, \, m_{\tilde{q}_j}), \tag{4.33}$$

where $\mathcal{G}$ is a dimensionless loop function given in Eq. (C.16), the j index sums over q-type scalar mass eigenstates, and k sums over the gauginos that couple to q and the Higgs, $SU(2)_L$ and $U(1)_Y$. The factor $r^q_i \equiv (1, 1, c_q)$ accounts for rotations in the fermion sector. A more explicit expression for y^u_{3i} is given in Eq. (C.21). To get the analogous y^u_{i3} contribution, we would replace $q \leftrightarrow u$, and only the bino would contribute.

The additional suppression for these terms comes from the initial product of rotation matrices. By the convention set below Eq. (4.31), the "4$^{\text{th}}$" gauge index corresponds to the $\tilde{Q}$ smessenger, while the i index goes from 1-3 over the MSSM fields. Thus, in the limit that the $\tilde{q}$ scalars are all mass degenerate, Eq. (4.33) vanishes exactly. That is not the generic situation, but since $\mathcal{G}$ has only logarithmic dependence on mass, there is still a large residual cancellation.[5] In practice, these "gaugino-Higgsino" loops are suppressed compared to any other one-loop contribution, and are even typically smaller than the parametrically two-loop contributions that generate 1^{st} generation masses. In the '13' and '31' entries though, they can have important subleading effects, and so we include them in our computations.

4.3.3.2 Wavefunction renormalization

It is well-known that in radiative flavor models we can get wavefunction renormalization at one loop from the same dynamics that generates masses. In our case, the SM quark superfields are renormalized by flavon-messenger, flavino-smessenger, and gaugino contributions shown in Fig. 4.9.[6] The Higgs also receives wavefunction renormalization from the fields which it has large couplings to, the messenger and 3^{rd} generation quark superfields. In our computations here we will neglect flavon and flavino loops in the down sector because they are suppressed by y_b^2 in the 2^{nd} and 3^{rd} generations. Effects involving d_4 and D can be larger, but since $m_b \ll m_{\text{sc}}$, to a good approximation we neglect kinetic mixing between the vector-like and the SM d

[5]Interestingly, the diagram formally diverges and requires regulation. In dimensional regularization, the $1/\epsilon$ pole replaces the finite loop function $\mathcal{G}$ in the divergent term. However, this removes any dependence on the mass eigenstates, and the rotation factor multiplies to zero. Thus, the contribution is actually finite and has no dependence on renormalization scale. There is, however, a renormalization scale dependent contribution to y_{33}^u coming from the q_3' portion of Q. In this case, the rotation matrix prefactor does not cancel upon summing over mass eigenstates. However, for our numerical analysis, we do not include one-loop corrections to y_t, and therefore drop this contribution as well as the finite one to y_{33}^u from Eq. (4.33).

[6]We also include the renormalization of u_3', q_3' due to Higgs superfields.

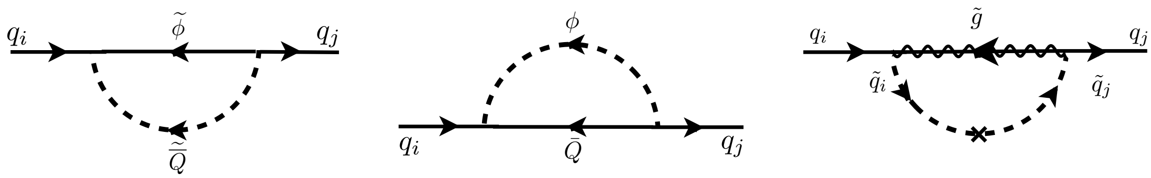


Figure 4.9: Diagrams that induce flavor-violating wavefunction renormalization for the fermions. The left two diagrams only contribute to the 2[nd] and 3[rd] generations, while the one on the right is present for all fermions.

quarks. We find that with the benchmark parameters presented in Sec. 4.3.4, including the heavy fermions in wavefunction renormalization only changed our SM quark predictions at the 1% level, and is thus below our working precision. Furthermore, calculating the one-loop shift in the mass of the d_4 and D-like quarks themselves is beyond our scope.

The one-loop wavefunction renormalization diagrams contribute to the usual Z_i factors for all the SM fields. Taking the up-type as an example, the Yukawa coupling $q\,\mathbf{y_u}\,u\,h$, is modified to

$$\mathbf{y_u} \to (\boldsymbol{Z_q})^{1/2}\,\mathbf{y_u}(\boldsymbol{Z_u})^{1/2}\,(Z_h)^{1/2}, \tag{4.34}$$

where $Z_i = 1 - \Sigma^i$ with Σ^i being the possibly divergent loop contributions whose one-loop expressions are given in App. C.2. For fermions, we will use the conventions and notation of [145]. We evaluate the divergent contributions at a common scale $\mu = 1$ PeV because that is where the heavy fields are integrated out. Errors induced from the fact that not all the heavy fields are exactly at 1 PeV are logarithmic in the change in mass and formally of higher loop order. In Eq. (4.34), we have bolded the terms which are matrices in flavor space. The $\boldsymbol{Z}$ factors for the quarks will in general have off diagonal components, particularly the gluino contribution because of the large squark mixing. Thus we see that wavefunction renormalization is a potentially important effect that not only rescales individual elements of the Yukawa matrices, but also rotates among them. The approximate size of the effects is an increase in the Yukawa couplings of 5-15%.

4.3.4 Standard Model Flavor Parameters

As laid out in Sec. 4.2, our model has the right parametric behavior to explain the generational hierarchy of the Standard Model fermion Yukawas and the parameters of the CKM matrix. Using the equations of Sec. 4.3.3 and App. C, we find a set of parameters that reproduce the SM quark masses, CKM angles, and phase to within 5% of their values listed in [1] for the former and [3] for the latter. We list the contributions computed in Table 4.5. Despite the close agreement we have obtained with the SM in the quark sector, it is important to stress that there are sources of uncertainty in our calculation discussed below at the level of $\mathcal{O}(15\%)$. The proximity of our current results to the SM values is meant as a demonstration of the control one has in recovering the SM. Thus, the inclusion of subleading corrections to the results we have obtained will likely provide no fundamental obstacle to precise recovery.

Table 4.5: Classes of contributions we include for up and down-type Yukawa matrix entries. Complete loop-level formulas are given in App. C, along with those for wavefunction renormalization, which we apply to all entries. The tree-level expressions are found in Eqs. (4.25) and (4.28) for y_t, y_b. For every entry listed, we include the same class of diagrams for its transpose.

y_{11}, y_{12}, y_{13}	gluino, gaugino-Higgsino
y_{22}, y_{23}	flavino, flavon, gluino, gaugino-Higgsino
y_{33}	tree-level

We now discuss the construction of our benchmark and its properties. The spectrum of new particles for these particular parameters is shown in Fig. 4.10. We generate the parameters of our flavon sector randomly. Scanning over $\mathcal{O}(1)$ values for dimensionless parameters, $\mathcal{O}(100-1000)$ TeV values for dimensionful ones, and taking phases in general to be $\mathcal{O}(1)$, we find a vacuum that is stable and breaks $U(1)_F$ symmetry with VEVs that can generate all SM masses. We then use the values of λ_U, $\bar{\lambda}_U$, λ_D, $\bar{\lambda}_D$ from Eqs. (4.2) and (4.9) (important for 3$^{\text{rd}}$ generation), as well as the $f^{q,u,d}$ couplings (2$^{\text{nd}}$ generation) and the squark soft masses (1$^{\text{st}}$ generation) plus Higgs and messenger μ, B_μ-terms as handles to recover the SM. If μ is too large, that

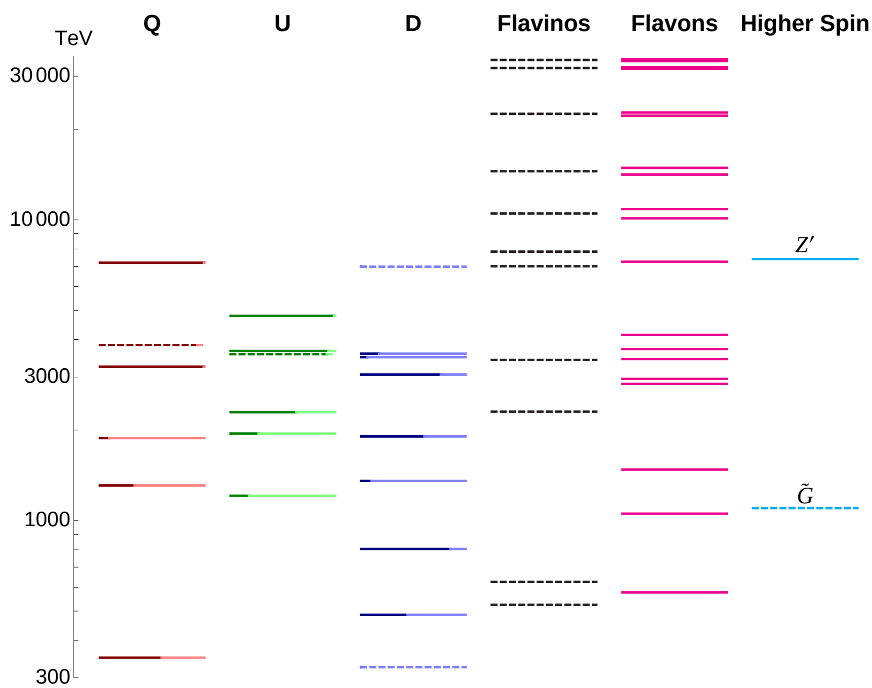


Figure 4.10: Spectrum of non-SM particles for parameters that closely reproduce the SM quark sector. Solid lines are bosons and dashed lines are fermions. Shading under Q, U, D indicates the portion of the mass eigenstate given by MSSM gauge eigenstates (dark) or messenger/d_4/$\bar{d}$ gauge eigenstates (light). We include only the mass mixing in this quantification. The flavino states also include the $U(1)_F$ gaugino which strongly mixes with them. The corresponding $U(1)_F$ gauge boson is shown under "Higher Spin," along with the gravitino. As discussed in the text and shown in Fig. 4.1, the gauginos are much lighter than all the fields here.

could potentially lead to deeper vacua that are color breaking [146–148], but we check that this is not a problem for our benchmark.

To have a viable thermal WIMP dark matter particle, we fix the wino mass at 3 TeV and obtain the gaugino spectrum ($m_{\tilde{B}} = 13.3$ TeV, $m_{\tilde{W}} = 3$ TeV, $m_{\tilde{g}} = 20.9$ TeV) as detailed in Sec. 4.3.1.[7] The gluino mass offers an additional means to control $m_{u,d}$. In the down sector we subject $f_d^{2,3}$ to the technically natural tuning at $\mathcal{O}(0.1)$. Dimensionful values were again $\mathcal{O}(100-1000)$ TeV. The only nontrivial constraint comes from kaon physics, further detailed in Sec. 4.4.1, and it favors having Q and D states $\gtrsim 1000$ TeV.

For comparison with the SM, we show the values we obtain for our Yukawas at the scale m_t in Fig. 4.11 compared to those depicted earlier for the SM (Fig. 1.1). In Fig. 4.12 we compare the CKM of our benchmark to that of the SM.[8] For the ten SM quark parameters shown here, the mean discrepancy with the SM is 4%, though as mentioned above, our results have an uncertainty of $\mathcal{O}(10-20\%)$. The leading effects that we are currently neglecting include: 1) Some dimensionless couplings are ≈ 1.3, leading to $\mathcal{O}(15\%)$ corrections at next-to-leading order; 2) Including wavefunction renormalization induces scale dependence. We evaluate at a common scale of 1000 TeV before integrating out all non-SM fields besides the gauginos. However, many of

[7] The $U(1)_F$ gaugino mixes strongly with the flavinos and has mass $\mathcal{O}(\text{PeV})$. We did not compute its full soft mass from anomaly mediation, but the flavino spectrum is highly insensitive to its detailed value if within a few orders of magnitude of the other gauginos.

[8] We have neglected the small running of the CKM parameters, which affects θ_{13} and θ_{23} most, at the level of a few percent [3]. We take that reference's SM values at 10 TeV computed in $\overline{\text{MS}}$ to compare to those in our model evaluated at 1000 TeV.

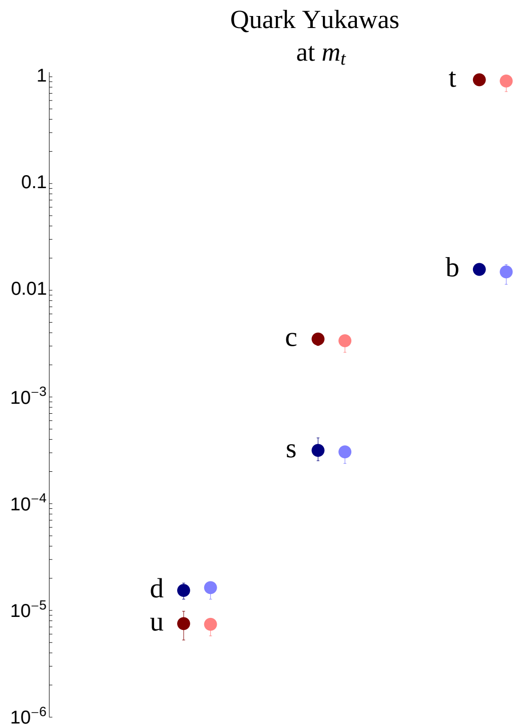


Figure 4.11: SM values from [1] (dark) and values obtained in our benchmark (light) at the scale m_t. Errors bars for the latter assume uniform shifts in Yukawas by $+15\%, -25\%$ at 1000 TeV, which accounts for a uniform uncertainty of $\pm 15\%$ in addition to a 10% decrease coming from choosing a renormalization scale that is lower than the mass of some of the states (see text). After applying these uncertainties in the UV, we run the Yukawas to m_t.

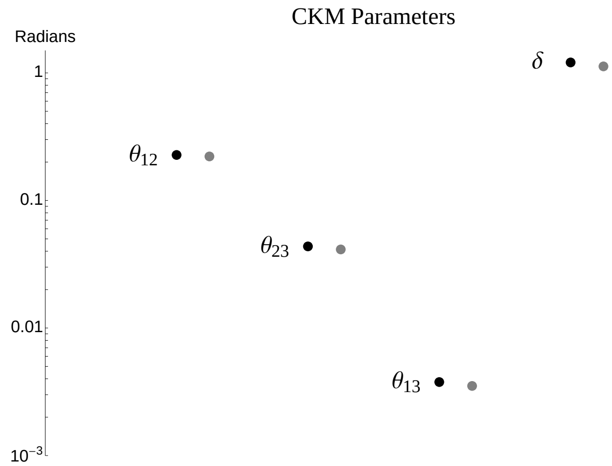


Figure 4.12: SM values from [3] (dark) and values obtained in our benchmark (light). Error bars are smaller than the dot size.

our messenger fields are above 1 PeV (with the heaviest at 7.2 PeV), and thus there are $\mathcal{O}(1)$ logs we are not currently resumming. Changing the renormalization scale from 1000 to 10,000 TeV, decreases our Yukawa values by $\mathcal{O}(10\%)$; 3) For $y_{33}^{u,d}$, we only include the tree-level values given in Eq. (4.25) for y_t and Eq. (4.28) for y_b. The one-loop corrections to these entries could shift them at the level of a few percent; 4) To compute quark masses and the CKM, we just take the 3x3 matrices in the up and down sectors. However, there are additional Yukawas with the messenger fermions, Q, U, D and d_4. There will also be kinetic mixing from one-loop wavefunction renormalization. Taking the values for our benchmark in the down sector, where we expect the effects to be strongest due to d_4, we found shifts in quark masses at the level of 1-2% for y_b and y_d, with y_s changing negligibly. Thus, we neglect this contribution as well; 5) For our gaugino loops that contribute strongly to 1st generation masses (*cf.* Fig. 4.5), we have treated the threshold correction due to messengers as a mass-insertion, even though these same messengers appear elsewhere dynamically in the loop. Including the full momentum-dependence of the one-loop correction to the gaugino propagator shifted our masses by $\lesssim 1\%$, which is beyond our precision and we thus ignore this effect.

Given the agreement in the quark sector, it would be an interesting exercise to reproduce charged leptons as well, something we did not attempt here. We would wish to maintain consistency with unification, so our $\lambda_{L,E}$ and $f^{l,e}$ couplings would need to be determined for the values we assigned to the quark-sector. The slepton soft masses and bino mass would offer independent means to control the lepton masses. In Sec. 4.5, we sketch a possible model extension that would generate neutrino masses and mixing angles. Before moving on we note that many of these sources of uncertainty affect the 3$^{\text{rd}}$ generation most strongly. Since those Yukawas are dominated by the tree-level contribution, we expect them to be the simplest to adjust once these additional effects are taken into account.

4.4 Experimental Constraints and Signatures

Detailed studies of the low-energy constraints on Mini-Split SUSY have been performed in [149–152]. The dominant processes are meson mixing, electric and chromoelectric dipole moments (CEDM), and lepton flavor violation. In addition to the MSSM fields previously studied, the messengers and flavons contribute to these observables. The latter leads to large deviations from the predictions of minimal Mini-Split SUSY for processes that only involve the 2^{nd} and 3^{rd} generations of the **10** fields, q, u and e.

The strongest bound comes from CP violation in $K-\overline{K}$ mixing, which requires that the squarks that have large couplings to the gluino and s and d quarks be heavier than a few hundred TeV. It is the only constraint we needed to compute in detail to test the viability of the benchmark in Sec. 4.3.4. Since it involves the 1^{st} generation, it is essentially a probe of the Mini-Split MSSM, though we account for the presence of messengers. While the limits from other observables are weaker, we discuss the contributions from flavon dynamics where they contribute strongly and present some detailed formulas in App. E. It will take many generations of future experiments to probe this sector. However, improved determination on the lattice of kaon parameters could provide evidence for one of the key ingredients of our model, the presence of anarchic squark mixing at several hundred TeV.

4.4.1 Meson Mixing

For the case of meson mixing, the MSSM effect is mediated by box diagrams with gluinos and squarks in the loops, but we neglect contributions with gluino mass insertions. Bounds are independent of the gluino mass as long as it is much lighter than the squarks. For our benchmark model, we check that the MSSM contribution does not run afoul of kaon constraints. We use the full mass eigenstate calculation of the squark-gluino box presented in [153] to account for the $\mathcal{O}(1)$ mixing among different squark gauge eigenstates and with the messenger sector. After matching to the relevant dimension-six operators at 1000 TeV, we run our Wilson coefficients at NLO to 2 GeV using the procedure outlined in [154], from which we also take numerical values for the bag parameters. For the benchmark in Sec. 4.3.4, we get

$$\begin{aligned} \epsilon_K^{\mathrm{NP}} &= 9.4 \times 10^{-5}\,, \\ \Delta m_K^{\mathrm{NP}} &= 2 \times 10^{-15}\,\mathrm{GeV}. \end{aligned} \tag{4.35}$$

Our contribution to Δm_K is safe by three orders of magnitude. The limit on ϵ_K^{NP} is $1{\times}10^{-3}$ [152, 154], so while our benchmark is safe, there are reasonable regions of parameter space in the model which are excluded by this observable. Thus, an improvement in ϵ_K^{SM} by an order of magnitude could be the best low energy way of probing the Mini-Split scenario.

In our model, there is a similar box diagram with flavino and messenger scalars in the loops. The only meson which is precisely measured and does not involve any 1^{st} generation quarks is the B_s. Therefore, the operator

$$\mathcal{O}_1 = (\bar{s}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L b), \tag{4.36}$$

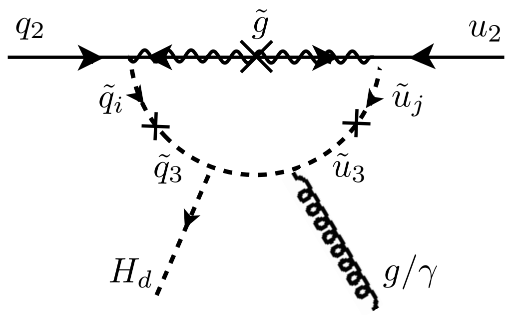
which contributes to B_s mixing will be modified by an $\mathcal{O}(1)$ amount relative to the MSSM, while the operators with other chiralities will be suppressed by powers of y_b.

We calculate the smessenger-flavino as well as the messenger-flavon diagrams that generate B_s mixing. Obtaining the Wilson coefficient for $\mathcal{O}_1$, we relate it to quantities in the B meson system following the approach of [155], using more recent numerical values from the lattice study in [156]. The detailed box diagram calculations are given in App. E. We get a contribution to the mass splitting $\Delta M_s = \mathcal{O}(10^{-20})$ GeV, compared to the SM value, $\Delta M_s = 1.2 \times 10^{-11}$ GeV and a shift in the total CP violating phase of $\mathcal{O}(10^{-11})$. Thus, experimental evidence is beyond the next generation of experiments.

4.4.2 (Chromo)Electric Dipole Moments

In the MSSM, (C)EDMs for all up-type quarks come from a one-loop diagram of the type shown on the left side of Fig. 4.13. This diagram has a gluino mass insertion, so it is proportional to $m_{\tilde{g}}/m_{\tilde{q}} \sim \mathcal{O}(10^{-2})$. These diagrams are comparable for u, c, and t if the squarks are anarchic in flavor space, but the strongest experimental bound comes from the up EDM. On the other hand, this model has one-loop diagrams with flavons and messengers going around the loop as shown on the right side of Fig. 4.13, as well as the supersymmetrized version with flavinos and smessengers. All the internal fields in this diagram have mass $\mathcal{O}(m_{\tilde{q}})$, so its effects are enhanced relative to the MSSM. Because the flavons only couple to the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ generation, these types of diagrams only induce (C)EDMs for top and charm.

The strongest bounds on these processes come from chromo-EDMs inducing con-



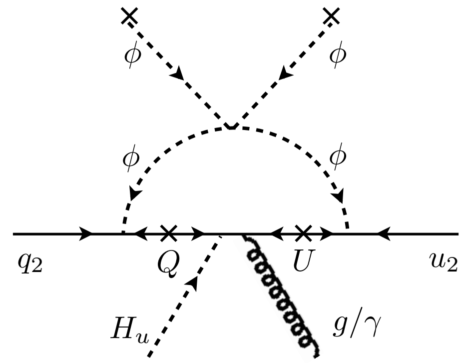


Figure 4.13: L: An example diagram of the MSSM gluino contribution to quark (C)EDMs. R: An example of the flavon contribution to quark (C)EDMs.

tributions to the neutron EDM. For the top quark, the bound was computed to be $|\tilde{d}_t| \lesssim 1/(100\text{ TeV})$ [157]. This computation uses the fact that there is a separation of scales between the top and Λ_{QCD} and runs operators down from the top mass to the QCD scale. We can approximate the bounds on the charm mass by ignoring the running effects besides the scale of α_s. Because integrating out a quark generates a finite contribution [158,159] to the Weinberg operator [160], integrating out a lighter quark will lead to a larger contribution to the neutron EDM. Furthermore, the gluon loop that generates this operator is larger because α_s is evaluated at m_c where it is much larger. We approximate $\alpha_s(m_c) \simeq \alpha_s(m_\tau) \simeq 0.35$ [161]. Because of these effects, the bounds on $\tilde{d}_c$ are much stronger than on top, and we find $|\tilde{d}_c| \lesssim (6 \times 10^5\,\text{TeV})^{-1}$, in rough agreement with the bound of $|\tilde{d}_c| \lesssim (2 \times 10^5\,\text{TeV})^{-1}$ from the more detailed study in [162]. These limits should be taken as accurate to within an order of magnitude because of the uncertainties on the hadronic matrix elements that go into the conversion of the Weinberg operator into the neutron EDM.

Taking the CEDM of the charm quark as an example, the low energy operator is of the form

$$-i\frac{\tilde{d}_c}{2} g_s\, \bar{c}\, \sigma^{\mu\nu}\, \gamma^5\, t^a c\, G^a_{\mu\nu}. \tag{4.37}$$

At the scale of electroweak symmetry breaking, this operator matches onto an operator involving the left-handed quark doublet, the right-handed singlet, and the Higgs. This can be seen from the fact that the tensor operator above flips the helicity of the quark, so it must involve a Higgs insertion. In the UV at the scale of SUSY breaking, this operator is generated by diagrams of the type shown in Fig. 4.13. As discussed above, the diagram on the right is the dominant contribution for the charm and top quarks, and we can estimate its size to be parametrically $\mathcal{O}(v/16\pi^2 m^2_{\text{mess}}) \simeq (10^{10}$ TeV$)^{-1}$ for $m_{\text{mess}} = 3000$ TeV.

We improve on the one-loop estimate by computing 1) the generalization of the right diagram in Fig. 4.13 to flavon and fermion mass-eigenstate basis, 2) an additional flavon-messenger diagram with no mass insertions (besides the SM Higgs VEV) proportional to $\bar{\lambda}_U$, and 3) the corresponding flavino-smessenger contribution. We project onto the Dirac structure of a chromoelectric dipole and obtain a numerical value by setting the Higgs to its VEV, even though we are formally matching at 1000 TeV.[9] Using the sign and normalization conventions of [152], we get $|\tilde{d}_t| = (4 \times 10^{12}$ TeV$)^{-1}$ and $|\tilde{d}_c| = (1.2 \times 10^{12}\,\text{TeV})^{-1}$, a bit below our estimate above since couplings

[9]Since the calculated values of $\tilde{d}_{c,t}$ are so far below current bounds, the effects of running to the fermion mass scale will not change our conclusions by much beyond an order of magnitude.

and mixing angles are accounted for, and far removed from near future sensitivity. We give the expressions for the flavon-sector loop contributions in App. E.

Analogues to the operator in Eq. (4.37) for the down and lepton sector will be suppressed by $\mathcal{O}(y_b) \sim \mathcal{O}(y_\tau)$. This is due to the small coupling of the $\bar{\mathbf{5}}$ to χ, as explained in Sec. 4.2.2. Therefore, the strange and bottom (C)EDMs are enhanced relative to the MSSM diagrams by $y_b m_{\tilde{q}}/m_{\tilde{g}} \sim 10$. For the strange quark, we take the formula from [163] to estimate an experimental bound of $\tilde{d}_s \lesssim (3 \times 10^6 \text{ TeV})^{-1}$, while the natural size in this model is $\tilde{d}_s \simeq y_b\, \tilde{d}_c \simeq (10^{11} \text{ TeV})^{-1}$. The CEDM for the b-quark can be computed in the same way. Thus, we see that the model is safe from (C)EDM measurement until several order of magnitude improvement is achieved.

4.4.3 Lepton Flavor Violation

Lepton flavor violation (LFV) is also a strong constraint on models with anarchic flavor structure, with $\mu \to e\gamma$ currently imposing the most stringent constraint in the MSSM. The diagrams for LFV have the same structure as those for EDMs shown in Fig. 4.13, so loops of flavons and messengers are enhanced by $\mathcal{O}(y_\tau m_{\tilde{q}}/m_{\tilde{B}}) \sim 10$ relative to the MSSM diagrams, but only for processes involving only 2$^{\text{nd}}$ and 3$^{\text{rd}}$ generation leptons. In this case, that means $\tau \to \mu\gamma$ and other rare τ decays are enhanced. In calculating the contribution of our model to charm and top quark (C)EDMs in Sec. 4.4.2, we also obtained comparable values for the flavor changing dipole operators. We can use our values in the up quark sector to estimate the contribution to the analogous lepton operator, which is given schematically as

$$\frac{e\, m_\tau}{16\pi^2 m_{\rm sc}^2}\, \bar{\tau}\, \sigma^{\mu\nu}\, \mu\, F_{\mu\nu}\,. \tag{4.38}$$

We expect this to be of similar order as the charm EDM. Naively, the numerator of the coefficient should be v since the Higgs insertion in Fig. 4.13 is on an internal line which has a large Yukawa coupling. On the other hand, there is a factor of y_τ coming from the coupling of the left-handed lepton to the flavon, so we can combine that with v to get m_τ. Taking into account $\alpha_{\rm EM}$, we estimate $\mathrm{BR}(\tau \to \mu\gamma) \sim 10^{-19}$. The current limit is $\mathcal{O}(10^{-8})$ with the possibility of one to two order of magnitude improvement at a future τ factory. Thus, this will unfortunately not provide a means to detect the flavor violation in our model in the near future. The contributions to ΔF $= 1$ processes in the quark sector are also significantly below the current experimental limits.

4.4.4 Proton Decay

The problem of proton decay is of a somewhat different nature than the other constraints. None of the terms in the renormalizable Lagrangian induce proton decay, but there are higher dimensional operators allowed by all the symmetries of the theory, such as the dimension five superpotential operator $qqq\ell$, that do. It has long been known that this is a problem in weak-scale SUSY [164–167]. Raising the scalar masses weakens the bounds, but recent analysis [149, 168, 169] has shown that this is still a problem, with the cutoff for dimension five operators needing to be higher than the Planck scale to make the proton live long enough.

One could imagine building a model in the spirit of this one such that $U(1)_F$ forbids the higher dimensional proton decay operators. These kinds of charge assignments tend to be anomalous,[10] so we go in a different direction here by noting that because proton decay is mediated by higher dimensional operators, it is clearly sensitive to the UV structure of the theory. Proton decay operators are generically generated by the physics of Grand Unification, but they need not be, as in the case of higher dimensional GUTs [170]. As proton decay is a generic problem for all SUSY models and in particular for SUSY GUTs, we simply assume that one of the solutions in the literature, such as [170], is active in the UV but has no impact on scales below the unification scale.

4.5 Conclusions

Supersymmetry has been a subject of intense study because of its many interesting theoretical and phenomenological features. As an extension of the Standard Model, it can solve the hierarchy problem, provide a natural WIMP dark matter candidate, and improve gauge coupling unification. The unfortunate lack of evidence for SUSY

[10] The $U(1)_F$ could be a spontaneously broken global symmetry with anomalous charges as in [44]. The anomaly will generate a mass for the Goldstone, but additional explicit breaking will likely be needed to raise it higher. Given the IR issues induced by adding this light state and the need to control corrections to the radiative story from having a merely approximate symmetry, we forego this possibility, though there may be a viable implementation.

at the LHC as well as the (fortunate) discovery of a Higgs with mass around 125 GeV has led to a reconsideration of weak-scale SUSY, with Mini-Split SUSY emerging as a framework with many intriguing features. In particular, with scalars around 1000 TeV and gauginos one loop factor lighter, the correct Higgs mass is obtained with dark matter and unification stories being comparably successful. SUSY would then only partially solve the hierarchy problem, leaving us with a meso-tuned picture of the universe.

In this chapter, we have explored how an additional feature of Mini-Split SUSY, the automatic solution of the SUSY flavor problem, can be used to address the SM flavor puzzle. In the Standard Model, there is no explanation for the peculiar structure of the masses and mixings of the quarks and leptons. Each generation is substantially lighter than the previous one, and the ratio of 3^{rd} to 2^{nd} generation masses appears remarkably similar to the ratio of 2^{rd} to 1^{nd} generation masses. Thus, one possible explanation of the SM flavor sector is that fermion masses are generated via a hierarchy of loops, with the 3^{rd} generation Yukawa coupling generated at tree level, the 2^{nd} at one loop, and the 1^{st} at two loops: a radiative explanation of flavor.

In the framework of Mini-Split, the scalars carry flavor quantum numbers and, unlike in weak-scale SUSY, there can be significant mixing between the different flavors of squarks. This mixing can be used in loops to generate the Yukawa couplings. Because new Yukawa couplings cannot be generated by loops in supersymmetric theories, the physics of flavor must be tied to the physics of SUSY breaking. Here we have built a model which radiates flavor around 1000 TeV, the scale which the Higgs mass points to. The full particle content of the model is given in Table A.1.

In the UV, this model forbids all Yukawa couplings to the SM matter with a new $U(1)$ symmetry under which the Higgs is charged, but all matter is neutral. SUSY breaking triggers the breaking of $U(1)_F$, and a Yukawa coupling is communicated via a rank 1 messenger sector. This allows only the 3$^{\text{rd}}$ generation to get a Yukawa coupling at tree level. The messengers can then generate additional Yukawas at one loop, but because of the size of the messenger sector, these loop contributions only affect the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ generation. Finally, there is the loop contribution from the sfermions, which is parametrically of two-loop order and involves all generations. This two-loop contribution is only big enough because there is large flavor mixing in the sfermion sector.

In addition to building a model and giving parametric estimates of the size of all the flavor parameters, we have also computed a detailed spectrum for the quark sector taking into account all leading effects including mixing and wavefucntion renormalization. We have shown that one can get agreement with all the SM flavor parameters to within 5% at a generic point in parameter space described in Sec. 4.3. We have also computed current constraints and found most of them to be trivially satisfied; however, the constraints from the Kaon system do exclude some of the parameter space. The phenomenology of this model is quite similar to Mini-Split SUSY, but in principle there are deviations in flavor observables involving the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ generation, such as B_s mixing.

In order to build a complete flavor model, neutrinos must also be included. One can think of neutrino masses as coming from the usual SM dimension five operator. Once the $U(1)_F$ is included, this operator can be generated by either of the following dimension 7 superpotential operators

$$\frac{1}{M_*^3}(\ell H_u)(\ell H_u)\bar{\chi}\,\phi \qquad\qquad \frac{1}{M_*^3}(\ell H_u)(\ell H_u)\xi^2\,, \tag{4.39}$$

where we have suppressed flavor indices. In this case, the neutrino masses will be given by $m_\nu \sim v^2\langle\bar{\chi}\rangle\langle\phi\rangle/M_*^3$ for the first operator, and the generalization is clear for the second. Here $v \simeq 174$ GeV is the electroweak scale. In the benchmark given in Sec. 4.3.4, the VEVs of the flavons are of order $100 - 1000$ TeV, so M_* can be as low as 100 PeV to reproduce the experimentally measured neutrino masses. This scale is somewhat above the scale of the model, but not dramatically. These operators can be UV completed with vector-like right-handed neutrinos with different F charges, but we leave this analysis including the computation of the neutrino mixing to future work.

Stepping back, we see that while the lack of evidence for SUSY at the LHC is beginning to close the door on weak-scale SUSY, a windows is perhaps opening into the Mini-Split paradigm. Through this window, we have envisioned a solution to the SM flavor puzzle, explaining the many hierarchies we have seen through the physics of radiative corrections. Only the ratio of the bottom to top quark masses is left unexplained, but this ratio is correlated with the size of the Cabibbo angle, giving unexpected agreement in both sectors. All other small numbers in the SM flavor sector are the result of loop corrections and a consequence of linear algebra. The theory does not need to distinguish different generations, yet it generates all the flavor hierarchies we observe in nature.

Appendix A

Field Content and $U(1)_F$ Gauge Symmetry

In this appendix we review the full field content and address some of the complications associated with introducing a new gauge group. The field content is given in Tables 4.1, 4.2, and 4.3, and we give the full field content here in Table A.1 for completeness. We begin by noting that all the fields in the theory transforming under $U(1)_F$ are vectorlike, so anomaly cancellation is satisfied trivially. This also allows us to write a supersymmetric mass term for all the fields that are not part of the MSSM. Since this term comes from the Giudice-Masiero mechanism, the mass is $\mathcal{O}(m_{\rm sc})$, so all the scalars and fermions given in Table A.1 are at the PeV scale. The one exception, of course, is the light Higgs, which is tuned to have a mass around 126 GeV.

Because the new gauge group is a $U(1)$, a Fayet-Iliopoulos [171] (FI) term is

Table A.1: The full particle content of our model in addition to that of the MSSM. We also give the charges under $\mathrm{U}(1)_F$, the SM gauge group, and R-parity. Note that the MSSM fields q, u, d, ℓ, e are neutral under $U(1)_F$ and negative under R_p.

Field	$\mathrm{U}(1)_F$	$SU(3) \times SU(2) \times U(1)$	R_p
$H_u,\ H_d$	∓ 2	$(\mathbf{1},\mathbf{2})_{1/2} + (\mathbf{1},\mathbf{2})_{-1/2}$	$+$
$Q,\ \bar{Q}$	± 1	$(\mathbf{3},\mathbf{2})_{1/6} + (\bar{\mathbf{3}},\mathbf{2})_{-1/6}$	$-$
$U,\ \bar{U}$	± 1	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3} + (\mathbf{3},\mathbf{1})_{2/3}$	$-$
$E,\ \bar{E}$	± 1	$(\mathbf{1},\mathbf{1})_{1} + (\mathbf{1},\mathbf{1})_{-1}$	$-$
$D,\ \bar{D}$	∓ 3	$(\bar{\mathbf{3}},\mathbf{1})_{1/3} + (\mathbf{3},\mathbf{1})_{-1/3}$	$-$
$L,\ \bar{L}$	∓ 3	$(\mathbf{1},\mathbf{2})_{-1/2} + (\mathbf{1},\mathbf{2})_{1/2}$	$-$
$\ell_4,\ \bar{\ell}$	0	$(\mathbf{1},\mathbf{2})_{-1/2} + (\mathbf{1},\mathbf{2})_{1/2}$	$-$
$d_4,\ \bar{d}$	0	$(\bar{\mathbf{3}},\mathbf{1})_{1/3} + (\mathbf{3},\mathbf{1})_{-1/3}$	$-$
$\phi_{1,2},\ \bar{\phi}_{1,2}$	± 1	$(\mathbf{1},\mathbf{1})_{0}$	$+$
$\chi_{1,2},\ \bar{\chi}_{1,2}$	∓ 3	$(\mathbf{1},\mathbf{1})_{0}$	$+$
$\xi,\ \bar{\xi}$	∓ 2	$(\mathbf{1},\mathbf{1})_{0}$	$+$

allowed by the gauge symmetry. Fortunately, a high scale FI term is inconsistent with supergravity [172] and will not be generated. We also assume that any intermediate dynamics between the Planck and PeV scales also does not generate an FI term. Another possibility arising from the abelian nature of the new group is kinetic mixing between hypercharge and $U(1)_F$ [173]. If hypercharge is embedded in a GUT, then this operator will be absent at the scale of GUT breaking, but it will be generated by loops of fields charged under both $U(1)$'s such as those in Table A.1. Because this is a loop effect, we will treat it as a perturbation.

Once $U(1)_F$ is broken, the gauge fields can be diagonalized by shifting the hypercharge field with component of the F gauge field. This has several effects, but all of them turn out be phenomenologically harmless in the context of Mini-Split SUSY. First, the hypercharged fields acquire some F charge. Because $U(1)_F$ is broken at such a high scale, this has no effect in present experiments. The D-term for $U(1)_F$ will also be shifted

$$D'_F = D_F + \epsilon D_Y, \tag{A.1}$$

where ϵ is the coefficient of the kinetic mixing operator. The potential goes as $(D'_F)^2$, which when expanded out contains two different effects. The first is a shift in the coefficient of the $U(1)_Y$ D-term by $\mathcal{O}(\epsilon^2)$. The second is an effective FI term for hypercharge coming from the cross term. Both of these modify the scalar potential for the hypercharged scalars, but they have no qualitative effect because all these scalars have large masses from SUSY breaking. Therefore, the effects of kinetic mixing on the D-term can be thought of as small corrections to the masses and quartics for these scalars.

Appendix B

Flavon Sector Details

Here we explain the field content and charges of the flavon sector and give a brief description of the potential minimization. In the UV, all SM Yukawa couplings are forbidden by $U(1)_F$, so in order to generate any Yukawas, we need flavons to get VEVs and spontaneously break $U(1)_F$. Thus we introduce a set of flavons ϕ, $\bar{\phi}$ with charges ± 1. This determines the charges of H_u, Q and U. In order to preserve anomaly cancellation and allow a μ-term for the Higgses, H_d must have opposite F charge to H_u. Because this is a supersymmetric theory and Yukawa couplings are superpotential operators, the down-Yukawa coupling must be to H_d, so we need a separate set of flavons, χ, $\bar{\chi}$, to generate down-type Yukawa couplings.

The analysis above shows that for the model to be viable, we need both ϕ and χ to get VEVs. Because of the structure of the potential, this turns out to be impossible without introducing additional flavons. Consider the potential for one set of ϕ, $\bar{\phi}$, χ,

$\bar{\chi}$, i.e. ignoring the fact that the flavons are doublets in the full model. The potential is given by

$$\begin{aligned} V &= m_\phi^2|\phi|^2 + m_{\bar{\phi}}^2|\bar{\phi}|^2 + m_\chi^2|\chi|^2 + m_{\bar{\chi}}^2|\bar{\chi}|^2 - (b_\phi\phi\bar{\phi} + c.c.) - (b_\chi\chi\bar{\chi} + c.c.) \\ &\quad + \frac{g_F^2}{2}\left(|\phi|^2 - |\bar{\phi}|^2 - 3|\chi|^2 + 3|\bar{\chi}|^2\right)^2 . \end{aligned} \tag{B.1}$$

The m_i^2 are real, and we can do field redefinitions so that the b's are positive and the VEVs are real and positive. In the supersymmetric limit, $m^2 = |\mu|^2 > 0$ and $b = 0$, so spontaneous $U(1)_F$ breaking is impossible. Once SUSY breaking effects are included, the soft masses can be negative and a b-term can be generated, so SUSY breaking can trigger $U(1)_F$ breaking.

We minimize the potential and get the following conditions:

$$\begin{aligned} 2m_\phi^2\phi - b_\phi\bar{\phi} + 2\phi D = 0 \qquad\qquad & 2m_\chi^2\chi - b_\phi\bar{\chi} - 6\chi D = 0 \\ 2m_{\bar{\phi}}^2\bar{\phi} - b_\phi\phi - 2\bar{\phi} D = 0 \qquad\qquad & 2m_{\bar{\chi}}^2\bar{\chi} - b_\phi\chi + 6\bar{\chi} D = 0, \end{aligned} \tag{B.2}$$

where $D = g_F^2\left(\phi^2 - \bar{\phi}^2 - 3\chi^2 + 3\bar{\chi}^2\right)$ is the D-term. Taking linear combinations of the left and right equations such that D cancels out gives us quadratic equations involving only the ϕ's or the χ's:

$$\bar{\phi}^2 - \frac{2}{b_\phi}\left(m_\phi^2 + m_{\bar{\phi}}^2\right)\phi\bar{\phi} + \phi^2 = 0 \qquad\qquad \bar{\chi}^2 - \frac{2}{b_\chi}\left(m_\chi^2 + m_{\bar{\chi}}^2\right)\chi\bar{\chi} + \chi^2 = 0, \tag{B.3}$$

which can be solved for the barred fields in terms of the unbarred ones, $\bar{\phi} = r_\phi\phi$, $\bar{\chi} = r_\chi\chi$, where r_ϕ and r_χ depend only on the parameters of the potential and not on the fields. Plugging this back into the minimization conditions, we have

$$
\begin{aligned}
(2m_\phi^2 - b_\phi r_\phi + 2D)\phi &= 0 & \qquad (2m_\chi^2 - b_\chi r_\chi - 6D)\chi &= 0 \\
(2m_{\bar\phi}^2 r_\phi - b_\phi - 2r_\phi D)\phi &= 0 & \qquad (2m_{\bar\chi}^2 r_\chi - b_\chi + 6r_\chi D)\chi &= 0.
\end{aligned}
\tag{B.4}
$$

Since we need nonzero VEVs for both the ϕ's and χ's, the expressions in parentheses must all simultaneously be zero.

We can get a new constraint by taking a linear combination of the first and third equations that eliminates D,

$$
(6m_\phi^2 - 3b_\phi r_\phi + 2m_\chi^2 - b_\chi r_\chi)\phi\chi = 0. \tag{B.5}
$$

Since the expression in parentheses is a function only of parameters and generically does not vanish, we are required to take either $\phi = 0$ or $\chi = 0$. Hence we conclude that with only the D-term quartic, either the ϕ's or the χ's will get VEVs, but not both. In the case of the full field content where ϕ and χ are doublets, there is less analytic control, but the conclusion still holds and either ϕ or χ will be stable at the origin.

In order to generate more VEVs, we need another potential term that will provide a quartic, so we must introduce another flavon pair, $\xi, \bar\xi$. Choosing the $U(1)_F$ charge of ξ to be -2, we can write down the superpotential operators given in Eq. (4.7), which give the following scalar potential

$$
\begin{aligned}
V_F = \lambda_\xi^2 \left(\phi^2\bar\chi^2 + \xi^2\bar\chi^2 + \xi^2\phi^2 + \text{ un-barred} \leftrightarrow \text{ barred}\right) \\
+2\mu\,\lambda_\xi(\bar\xi\phi\bar\chi + \xi\bar\chi\bar\phi + \xi\phi\chi + \text{ un-barred} \leftrightarrow \text{ barred}),
\end{aligned}
\tag{B.6}
$$

where for notational simplicity, we have taken a common supersymmetric mass μ for all the flavons (and continue to assume that all parameters are real). The potential is now quite complicated, but there are generic regions in parameter space where all flat directions are stabilized because of the extra quartic and the origin for all fields is destabilized. Numerical study confirms that generically, if one field gets a VEV, then all of them will.

In the above discussion, we chose the charge of ξ to be -2. This turns out to be the unique choice for a viable model. In order to understand this, we examine the most general $U(1)$ that arises from our democratic treatment of SM fields and that is allowed by the flavon couplings required to obtain tree-level Yukawa couplings. We can parametrize the charges of the SM fields under this U(1) as $X_{10}^{SM} = a$, $X_{\bar{5}}^{SM} = b$, and $X_{H_u} = -X_{H_d} = c$. The messenger-Higgs couplings then imply messenger charges of $X_{10}^{\text{mess.}} = -c/2$ and $X_{\bar{5}}^{\text{mess.}} = 3c/2$. These then fix the flavon charges to be $X_\phi = -c/2 - a$, and $X_\chi = 3c/2 - b$, so there are three independent $U(1)$ symmetries which allow the Yukawa couplings and mass terms of the theory. Our $U(1)_F$ flavor symmetry corresponds to the case $a = b = 0$. A second independent $U(1)$ can be parametrized by $a = c = 0$, under which $X_\chi = -b$ based on the above. Since generically the flavons all get VEVs, this global $U(1)$ would be spontaneously broken and would yield a highly problematic massless Goldstone. Rounding out the basis of $U(1)$'s is one under which the flavons are uncharged and is therefore unbroken. Demanding that the flavons are uncharged leads to the conditions $b = -3a$ and $c = -2a$; this charge assignment is related by a hypercharge rotation to $B - L$, and remains unbroken.

In order to get a viable spectrum with no Goldstone bosons, we need to explicitly break the second $U(1)$ while leaving $B - L$ and $U(1)_F$ unbroken. By adding an additional vectorlike flavon pair, we gain an additional unconstrained charge. Therefore, in order to break the second $U(1)$, we must assign $U(1)_F$ charges to the new flavon such that two different types of interactions can be written down for the new flavon so that no charge assignment under the second $U(1)$ will be consistent. This uniquely determines the $U(1)_F$ charge to be ∓ 2 because that is the only charge that allows us to write both $\phi\phi\xi$ and $\bar{\phi}\bar{\chi}\xi$. These are the interactions found in Eq. (4.7), which are needed for the dominant loop contribution to the 2^{nd} generation masses. In particular, the interaction with two ϕ's is needed for the charm mass, and the one with ϕ and χ is needed for the strange and muon mass. Therefore, we see that the charge assignment we have chosen for the flavons is not only necessary to get a viable spectrum of flavons, it is also crucial for generating the correct Standard Model Yukawa couplings.

Appendix C

Constructing the Yukawa Matrices

C.1 Radiative Yukawa Generation

In this appendix we give the formulas for the loop effects used to generate the SM Yukawa matrices. The "33" elements are generated at tree-level and are given by y_t in Eq. (4.25) and y_b in Eq. (4.28), and we do not consider loop-level shifts to $y_{33}^{u,d}$ because they are below our numerical precision. We now proceed to fill out the remainder of the Yukawa matrices with radiative contributions that we list in order of decreasing size.

a. Flavino/Flavon:

The dominant contribution to y_{ij} for $(i, j) = 2, 3$ comes from the flavino loop, which effectively sets the size of the 2^{nd} generation masses and is illustrated on the left-hand-side of Fig. 4.4. In the up sector

$$y^u_{ij} = \frac{1}{16\pi^2} r^u_i r^q_j \, m_{\tilde{\phi}_k} \widetilde{F}^u_{ik} \widetilde{F}^q_{jk} \Gamma^{5l^*}_{\tilde{q}} \Gamma^{5m^*}_{\tilde{u}} \, H^{QU}_{lm} \, \mathcal{F}(m_{\tilde{\phi}_k},\, m_{\tilde{q}_l},\, m_{\tilde{u}_m}), \tag{C.1}$$

$$\mathcal{F}(m_1,\, m_2,\, m_3) = \frac{m_1^2 \left(m_2^2 \log \frac{m_1^2}{m_2^2} - m_3^2 \log \frac{m_1^2}{m_3^2} \right) + m_2^2 m_3^2 \log \frac{m_2^2}{m_3^2}}{\left(m_1^2 - m_2^2\right)\left(m_1^2 - m_3^2\right)\left(m_2^2 - m_3^2\right)}. \tag{C.2}$$

Here r accounts for the fact that the original quark fields appearing at the vertices to which the external lines connect might not be mass eigenstates, for example

$$r^q = (1, 1, c_q, -s_q), \tag{C.3}$$

where the first two components are trivial because the first two generations do not mix with messengers and the last two are the q_3' projections of the gauge eigenstates q_3 and Q, respectively, and similarly for u. In Eq. (C.1), i, j simply take values 2-3, and thus here we only need the first three components of $r^{q,u}$, but we present the complete vector which will be used below. $\widetilde{F}_{ij}$ is the coupling of the i^{th} quark flavor to the j^{th} flavino mass eigenstate, for example

$$\widetilde{F}^q_{ij} = F^q_{ik} \Gamma^{kj}_{\tilde{\Phi}}, \; k = 1, 2 \;, \tag{C.4}$$

and we use a similar definition for $\tilde{\tilde{f}}_k$ in the down sector. The factor H^{QU}_{ij} is the coupling of the i^{th} $\tilde{q}$ and j^{th} $\tilde{u}$ mass eigenstates to the light Higgs, which arises from summing over all six triple scalar couplings involving Q, U smessengers and Higgses:

$$\begin{aligned} H^{QU}_{ij} \;=\; & -\mu_H \left(\lambda_U \cos\beta \, \Gamma^{4i}_{\tilde{q}} \Gamma^{4j}_{\tilde{u}} - \bar{\lambda}^*_U \sin\beta \, \Gamma^{5i}_{\tilde{q}} \Gamma^{5j}_{\tilde{u}} \right) + \mu_Q \left(\lambda_U \sin\beta \, \Gamma^{5i}_{\tilde{q}} \Gamma^{4j}_{\tilde{u}} - \bar{\lambda}^*_U \cos\beta \, \Gamma^{4i}_{\tilde{q}} \Gamma^{5j}_{\tilde{u}} \right) \\ & + \mu_U \left(\lambda_U \sin\beta \, \Gamma^{4i}_{\tilde{q}} \Gamma^{5j}_{\tilde{u}} - \bar{\lambda}^*_U \cos\beta \, \Gamma^{5i}_{\tilde{q}} \Gamma^{4j}_{\tilde{u}} \right). \end{aligned} \tag{C.5}$$

Because the fermion diagonalization is more involved, the down sector actually has two types of flavino diagrams. The first is the analogue of the up sector diagram:

$$y_{ij}^d = \frac{1}{16\pi^2} S_{in} r_n^d r_j^q \, m_{\tilde{\phi}_k} \widetilde{F}_{nk}^d \widetilde{F}_{jk}^q \Gamma_{\tilde{q}}^{5l^*} \Gamma_{\tilde{d}}^{7m^*} H_{lm}^{QD} \mathcal{F}(m_{\tilde{\phi}_k}, \, m_{\tilde{q}_l}, \, m_{\tilde{d}_m}), \tag{C.6}$$

where $l = 1 - 5$ and $m = 1 - 7$,

$$\begin{aligned} H_{ij}^{QD} \;&=\; \mu_H \left(\lambda_D \sin\beta \, \Gamma_{\tilde{q}}^{5i} \Gamma_{\tilde{d}}^{5j} + \bar{\lambda}_D^* \cos\beta \, \Gamma_{\tilde{q}}^{5i} \Gamma_{\tilde{d}}^{7j} \right) - \mu_Q \left(\lambda_D \cos\beta \, \Gamma_{\tilde{q}}^{5i} \Gamma_{\tilde{d}}^{7j} + \bar{\lambda}_D^* \sin\beta \, \Gamma_{\tilde{q}}^{5i} \Gamma_{\tilde{d}}^{7j} \right) \\ &- \mu_D \left(\lambda_D \cos\beta \, \Gamma_{\tilde{q}}^{5i} \Gamma_{\tilde{d}}^{7j} + \bar{\lambda}_D^* \sin\beta \, \Gamma_{\tilde{q}}^{5i} \Gamma_{\tilde{d}}^{7j} \right), \end{aligned} \tag{C.7}$$

$$r^d = (1, 1, \Gamma_d^{11}, \Gamma_d^{21}, \Gamma_d^{31}). \tag{C.8}$$

The matrix S_{ij} sums the relevant vertex over all possible gauge eigenstate fermion fields j overlapping with mass eigenstate field i

$$S_{ij} = \begin{cases} \delta_{ij}, & i \neq 3 \\ 0, & i = 3, \, j < 3 \\ 1, & i = 3, \, j \geq 3 \end{cases} \tag{C.9}$$

One could extend the definitions of r^d and S_{ij} to include the higher generation mass eigenstates (d_4', D'), but we do not consider the effects of mixing from these states, as we found them to be 1% effects. Furthermore, the renormalization of the heavy fermion masses themselves is beyond our scope. In Eq. (C.6), we take $n = 1 - 4$. Since we also have the coupling $\bar{d} D \bar{\chi}$ in the down sector, there is also an additional contribution for $i = 3$, since we can take the d_3' component of D:

$$y_{3j}^d = \frac{1}{16\pi^2} r_5^d r_j^q \, m_{\tilde{\phi}_k} \widetilde{F}_{jk}^q \tilde{\bar{f}}_k \, \Gamma_{\tilde{q}}^{5l^*} \Gamma_{\tilde{d}}^{6m^*} H_{lm}^{QD} \mathcal{F}(m_{\tilde{\phi}_k}, \, m_{\tilde{q}_l}, \, m_{\tilde{d}_m}). \tag{C.10}$$

Next we consider the contributions from flavon loops; an example appears on the right-hand-side of Fig. 4.4. There are two classes of flavon diagrams: one involves a mass insertion on each messenger line and the other does not. We begin with the mass-insertion type, and again start with the simpler up sector:

$$\begin{aligned} y^u_{ij} &= \frac{\lambda_U M_{Q'} M_{U'}}{32\pi^2} c_u c_q\, r^u_i r^q_j \\ &\times \left(\widehat{F}^{(u)Re}_{ik} + i\widehat{F}^{(u)Im}_{ik}\right)\left(\widehat{F}^{(q)Re}_{jk} + i\widehat{F}^{(q)Im}_{jk}\right)\mathcal{F}(m_{\phi_k},\, M_{Q'},\, M_{U'}), \end{aligned} \tag{C.11}$$

where $M_{Q'}$, $M_{U'}$ are the messenger mass eigenvalues, i.e. $M_{Q'} = \sqrt{\mu_Q^2 + |F^q_{3i}\,\phi_i|^2}$ and similarly for U', and $\widehat{F}^{Re}_{ij}$, $\widehat{F}^{Im}_{ij}$ is the coupling of the i^{th} quark flavor to the j^{th} flavon mass eigenstate, obtained by summing over the real and imaginary parts of the appropriate gauge eigenstate flavon doublet, respectively; for example

$$\begin{aligned} \widehat{F}^{(q)Re}_{ij} &= F^q_{ik}\Gamma^{kj}_{\Phi},\ k = 1, 2 \\ \widehat{F}^{(q)Im}_{ij} &= F^q_{ik}\Gamma^{kj}_{\Phi},\ k = 11, 12 \end{aligned} \tag{C.12}$$

and similarly for u, d, and $\hat{\bar{f}}_j$ in the down sector. The down sector again has two types of diagrams that correspond to this, with the first exactly analogous:

$$\begin{aligned} y^d_{ij} &= -\frac{\lambda_D M_{Q'} M^l_{D'}}{32\pi^2} S_{im} r^d_m r^q_j\, \Gamma^{3,l+1}_d \Gamma^{2,l}_{\bar{d}}\, c_q \\ &\times \left(\widehat{F}^{(d)Re}_{mk} + i\widehat{F}^{(d)Im}_{mk}\right)\left(\widehat{F}^{(q)Re}_{jk} + i\widehat{F}^{(q)Im}_{jk}\right)\mathcal{F}(m_{\phi_k},\, M_{Q'},\, M^l_{D'}), \end{aligned} \tag{C.13}$$

where the sum over $l = 1, 2$ accounts for both heavy d-type fermions, and $m = 1 - 4$.

The second type again involves the additional $\bar{f}$ coupling:

$$\begin{aligned} y^d_{3i} &= -\frac{\lambda_D M_{Q'} M^k_{D'}}{32\pi^2} r^d_5 r^q_i \, \Gamma^{3,k+1}_d \Gamma^{1,k}_{\bar{d}} c_q \\ &\quad \times \left(\widehat{F}^{(q)Re}_{ij} + i\widehat{F}^{(q)Im}_{ij} \right) \left(\hat{\bar{f}}^{Re}_j + i\hat{\bar{f}}^{Im}_j \right) \mathcal{F}(m_{\phi_j},\, M_{Q'},\, M^k_{D'}), \end{aligned} \tag{C.14}$$

The second class of flavon diagrams does not have mass insertions, so these involve the $\bar{\lambda}_U$ and $\bar{\lambda}_D$ couplings. Although the loop integral is formally divergent, for entries other than $y^{u,d}_{33}$ we need only consider the resulting finite log terms that are a function of the flavon masses. This is because the F-coupling piece vanishes when summed over all flavon mass eigenstates for $(i, j) \neq (3, 3)$, a consequence of the fact that the first two generations do not couple to the Higgs at tree level. For the up sector

$$y^u_{ij} = -\frac{\bar{\lambda}^*_U}{16\pi^2} r^u_i r^q_j \cos\beta \left(\widehat{F}^{(u)Re}_{ik} + i\widehat{F}^{(u)Im}_{ik} \right) \left(\widehat{F}^{(q)Re}_{jk} + i\widehat{F}^{(q)Im}_{jk} \right) \mathcal{G}(M_{Q'},\, M_{U'},\, m_{\phi_k}), \tag{C.15}$$

$$\mathcal{G}(\mu,\, M,\, m) = \frac{m^4 \left(M^2 - \mu^2\right) \log\left(\frac{m^2}{\mu^2}\right) - M^4 \left(m^2 - \mu^2\right) \log\left(\frac{M^2}{\mu^2}\right)}{2\left(m^2 - \mu^2\right)\left(m^2 - M^2\right)\left(M^2 - \mu^2\right)}. \tag{C.16}$$

The analogous down sector formula is

$$y^d_{ij} = -\frac{\bar{\lambda}^*_D}{16\pi^2} S_{im} r^d_m r^q_j \sin\beta \, \Gamma^{2,l}_{\bar{d}} \Gamma^{2,l^*}_{\bar{d}} \left(\widehat{F}^{(d)Re}_{mk} + i\widehat{F}^{(d)Im}_{mk} \right) \left(\widehat{F}^{(q)Re}_{jk} + i\widehat{F}^{(q)Im}_{jk} \right) \mathcal{G}(M_{Q'},\, M^l_{D'},\, m_{\phi_k}) \tag{C.17}$$

where $m = 1 - 4$, and the diagram generated from the $\bar{f}$ coupling is

$$y^d_{3i} = -\frac{\bar{\lambda}^*_D}{16\pi^2} r^d_5 r^q_i \sin\beta \, \Gamma^{1,k}_{\bar{d}} \Gamma^{2,k^*}_{\bar{d}} \left(\widehat{F}^{(q)Re}_{ij} + i\widehat{F}^{(q)Im}_{ij} \right) \left(\hat{f}^{Re}_j + i\hat{f}^{Im}_j \right) \mathcal{G}(M_{Q'},\, M^k_{D'},\, m_{\phi_j}) \tag{C.18}$$

b. Gluino:

The gluino loop, appearing in Fig. 4.5, is the dominant contribution to the entries in the first row and column of the Yukawa matrices, and controls the size of the 1^{st} generation mass. For the up sector,

$$y^u_{ij} = -\frac{\alpha_3}{2\pi} S_{im} r^u_m S_{jn} r^q_n \Gamma^{mk}_{\tilde{u}} \Gamma^{nl}_{\tilde{q}} H^{QU}_{kl} \mathcal{F}(m_{\tilde{g}},\, m_{\tilde{u}_k},\, m_{\tilde{q}_l}), \tag{C.19}$$

where $m, n = 1-4$. For the down sector,

$$y^d_{ij} = -\frac{\alpha_3}{2\pi} S_{im} r^d_m S_{jn} r^q_n \Gamma^{mk}_{\tilde{d}} \Gamma^{nl}_{\tilde{q}} H^{QD}_{kl} \mathcal{F}(m_{\tilde{g}},\, m_{\tilde{d}_k},\, m_{\tilde{q}_l}), \tag{C.20}$$

where $m = 1-5$ and $n = 1-4$. There are also analogous diagrams with a bino.

Approximating the threshold correction from messengers to the gaugino masses as a mass insertion is not strictly accurate at a PeV because other fields in this diagram are at that scale. A more precise way to calculate would be be to blow up the mass insertion and include the two-loop effects of the messengers. This can be improved further still by resumming the effects of the thresholds that generate the gluino mass, and then including the full momentum dependence in the gluino propagator. We find that these effects only modify the contribution to the Yukawa coupling by $\mathcal{O}(1\%)$, so for computing our benchmark we use the simpler analytic formulas above.

c. Gaugino-Higgsino

These diagrams, shown in Fig. 4.8, have already been discussed in Sec. 4.3.3.1. Here, we just present the complete formulas:

$$
\begin{aligned}
y^u_{i3} &= \frac{\alpha_Y}{\pi} Q^Y_u Q^Y_{H_u}\, \lambda_U s_q\, S_{ik}\, r^u_k\, \Gamma^{4j}_{\tilde{u}}\, \Gamma^{kj^*}_{\tilde{u}}\, \mathcal{G}(\mu_H,\, m_{\tilde{B}},\, m_{\tilde{u}_j}),\; k = 1-4 \\
y^d_{i3} &= -\frac{\alpha_Y}{\pi} Q^Y_d Q^Y_{H_d}\, \lambda_D s_q\, S_{ik}\, r^d_k\, \Gamma^{5j}_{\tilde{d}}\, \Gamma^{kj^*}_{\tilde{d}}\, \mathcal{G}(\mu_H,\, m_{\tilde{B}},\, m_{\tilde{d}_j}),\; k = 1-5 \\
y^u_{3i} &= \lambda_U s_u\, S_{ik}\, r^q_k\, \Gamma^{4j}_{\tilde{q}}\, \Gamma^{kj^*}_{\tilde{q}} \left[\frac{\alpha_Y}{\pi} Q^Y_q Q^Y_{H_u}\, \mathcal{G}(\mu_H,\, m_{\tilde{B}},\, m_{\tilde{q}_j}) - \frac{\alpha_2}{\pi} C_F\, \mathcal{G}(\mu_H,\, m_{\tilde{W}},\, m_{\tilde{q}_j})\right], \\
y^d_{3i} &= \lambda_D \Gamma^{21}_d\, S_{ik}\, r^q_k\, \Gamma^{4j}_{\tilde{q}}\, \Gamma^{kj^*}_{\tilde{q}} \left[-\frac{\alpha_Y}{\pi} Q^Y_q Q^Y_{H_d}\, \mathcal{G}(\mu_H,\, m_{\tilde{B}},\, m_{\tilde{q}_j}) \right. \\
&\quad \left. + \frac{\alpha_2}{\pi} C_F\, \mathcal{G}(\mu_H,\, m_{\tilde{W}},\, m_{\tilde{q}_j})\right], \qquad \text{(C.21)}
\end{aligned}
$$

where $k = 1-4$ in the last two equations above, Q^Y is the field hypercharge, and C_F is the quadratic Casimir for the group, which is equal to 3/4 and 4/3 for $SU(2)_L$ and $SU(3)_C$, respectively.

C.2 Wavefunction Renormalization

Our procedure is outlined in Sec. 4.3.3.2, where the generic diagram appears and the contribution from the flavon sector is discussed. There are also loops involving gluino/squarks, Higgsino/squarks, and Higgs/quarks. We include all of these in the renormalization of the q and u fields, but retain only the gluino contribution for d, since the other loops are y_b suppressed, or involve kinetic mixing, which we found to be an $\mathcal{O}(1\%)$ effect on our SM model prediction, and thus neglect. In addition, since only the 3rd generation couples to the Higgs at tree level, the diagrams involving the Higgs multiplet only contribute to the "33" entries. Unlike the contributions to

$y_{33}^{u,d}$, we do include wavefunction renormalization of the 3rd generation, as the Källén-Lehmann representation theorem along with the positivity of quantum mechanics determines that all such contributions will increase the 3rd generation masses [174]. Taken together, we find shifts upwards of 10%, and thus we cannot neglect them. We use the notation introduced in the previous section and take a renormalization scale $Q = 1000$ TeV, the common scale at which the heavy states are integrated out.

a. Gluino:

$$\Sigma_{ij}^{q} = \frac{\alpha_3}{2\pi} C_F S_{il} r_l^{q^*} S_{jm} r_m^q \Gamma_{\tilde{q}}^{lk} \Gamma_{\tilde{q}}^{mk^*} \mathcal{W}(m_{\tilde{g}},\, m_{\tilde{q}_k},\, Q),\; l,m = 1-4, \tag{C.22}$$

$$\mathcal{W}(M,\, m,\, Q) = \frac{M^4 \log\left(\frac{M^2}{Q^2}\right)}{2(M^2-m^2)^2} + \frac{m^2\,(m^2-2M^2)\log\left(\frac{m^2}{Q^2}\right)}{2(M^2-m^2)^2} + \frac{m^2-3M^2}{4(M^2-m^2)}. \tag{C.23}$$

Analogous formulas hold for u and d; in the case of d, we take $l,m = 1-5$. There is also a wino contribution to q.[1]

b. Flavino:

$$\Sigma_{ij}^{q} = \frac{1}{16\pi^2} r_i^{q^*} r_j^q \widetilde{F}_{ik}^{q^*} \widetilde{F}_{jk}^{q} \Gamma_{\tilde{q}}^{5l^*} \Gamma_{\tilde{q}}^{5l} \mathcal{W}(m_{\tilde{\phi}_k},\, m_{\tilde{q}_l},\, Q), \tag{C.24}$$

[1] We neglect the contribution from the $U(1)_F$ gauge supermultiplet, which only changes the "33" entry from the overlap of the 3rd generation massless eigenstate with the messenger gauge eigenstate. This is expected to be a $\mathcal{O}(1\%)$-level effect.

and similarly for u.

c. Flavon:

$$\Sigma^q_{ij} = \frac{1}{32\pi^2} r_i^{q^*} r_j^q \left(\widehat{F}_{ik}^{(q)Re} + i\widehat{F}_{ik}^{(q)Im}\right)^* \left(\widehat{F}_{jk}^{(q)Re} + i\widehat{F}_{jk}^{(q)Im}\right) \mathcal{W}(M_{Q'},\, m_{\phi_k},\, Q), \quad \text{(C.25)}$$

and analogously for u.

d. Higgsino:

$$\Sigma^q_{33} = \frac{|s_q|^2}{16\pi^2} \left(|\lambda_U|^2 \Gamma_{\tilde{u}}^{4i^*} \Gamma_{\tilde{u}}^{4i} \mathcal{W}(\mu_H,\, m_{\tilde{u}_i},\, Q) + |\lambda_D|^2 \Gamma_{\tilde{d}}^{5i^*} \Gamma_{\tilde{d}}^{5i} \mathcal{W}(\mu_H,\, m_{\tilde{d}_i},\, Q)\right), \quad \text{(C.26)}$$

where the first term arises from putting $\tilde{H}_u$ in the loop and the second has $\tilde{H}_d$. The contribution to Σ^u_{33} consists of only the $\tilde{H}_u$ piece and is obtained by taking $\tilde{u} \to \tilde{q}$.

e. Higgs:

Since our renormalization scale is far above the scale of electroweak symmetry breaking, $SU(2)$ is approximately unbroken. Therefore, the tuning in the Higgs sector produces one light doublet that includes the SM Higgs, and one heavy doublet with degenerate scalars of mass m_A. In other words, the two Higgs doublets are in the extreme decoupling limit of the MSSM. The light doublet is given by $H_1 = -\cos\beta(i\sigma_2)H_d^* + \sin\beta H_u$, and the heavy doublet by $H_2 = \sin\beta(i\sigma_2)H_d^* + \cos\beta H_u$.

The effects here describe loops with at least one heavy field (light Higgs or SM fermion masses are approximated to be 0 in the calculation), with purely light field effects taken into account in the RG evolution of the Yukawas from the high scale down to the weak scale.

The effects of a heavy Higgs and fermion loop are given by

$$\begin{aligned}
\Sigma^q_{33} &= \frac{|s_q|^2}{16\pi^2}\left[|\lambda_U|^2\cos^2\beta\left(|s_u|^2\,\mathcal{W}(0,\,m_A,\,Q)+c_u^2\mathcal{W}(M_{U'},\,m_A,\,Q)\right)\right. \\
&\quad \left.+\,|\lambda_D|^2\sin^2\beta\left(\Gamma_d^{31}\Gamma_d^{31^*}\mathcal{W}(0,\,m_A,\,Q)+\Gamma_d^{3i}\Gamma_d^{3i^*}\mathcal{W}(M^i_{D'},\,m_A,\,Q)\right)\right] \\
\Sigma^u_{33} &= \frac{|\lambda_U|^2}{16\pi^2}|s_u|^2\cos^2\beta\left[|s_q|^2\,\mathcal{W}(0,\,m_A,\,Q)+c_q^2\,\mathcal{W}(M_{Q'},\,m_A,\,Q)\right], \qquad \text{(C.27)}
\end{aligned}$$

while the effects of a light Higgs and a heavy messenger are given by

$$\begin{aligned}
\Sigma^q_{33} &= \frac{|s_q|^2}{16\pi^2}\left[|\lambda_U|^2\sin^2\beta\,c_u^2\mathcal{W}(M_{U'},\,0,\,Q)+|\lambda_D|^2\cos^2\beta\,\Gamma_d^{3i}\Gamma_d^{3i^*}\mathcal{W}(M^i_{D'},\,0,\,Q)\right] \\
\Sigma^u_{33} &= \frac{|\lambda_U|^2}{16\pi^2}|s_u|^2\,c_q^2\,\sin^2\beta\,\mathcal{W}(M_{Q'},\,0,\,Q). \qquad \text{(C.28)}
\end{aligned}$$

Appendix D

Messenger Threshold Corrections to Gaugino Masses

Here we generalize the discussion of Sec. 4.3.1 to fully account for mixing. We organize the bookkeeping by introducing tensors V^M and $W^{\bar{M}}$, which characterize the possible vertices. These are relatively simple for Q and U, since $\bar{Q}$ and $\bar{U}$ are already mass eigenstates and there is only one nonzero eigenvalue:

$$\begin{aligned} V^Q &= (\Gamma_{\tilde{q}}^{3i^*} s_q^*, \Gamma_{\tilde{q}}^{4i^*} c_q) \\ W^{\bar{Q}} &= \Gamma_{\tilde{q}}^{5i}, \end{aligned} \tag{D.1}$$

and analogously for U. Here V is a matrix, with the row denoting whether q_3 or Q is at the vertex, and we need to project out the component corresponding to the heavy fermionic mass eigenstate. Although we label them by D, the tensors for the down type messengers actually combine what were previously separate contributions from the d_4, $\bar{d}$ and D, $\bar{D}$ pairs:

$$
\begin{aligned}
V^D &= \left(\left(\Gamma_{\tilde{d}}^{3i^*}\Gamma_d^{12}, \Gamma_{\tilde{d}}^{3i^*}\Gamma_d^{13}\right), \left(\Gamma_{\tilde{d}}^{4i^*}\Gamma_d^{22}, \Gamma_{\tilde{d}}^{4i^*}\Gamma_d^{23}\right), \left(\Gamma_{\tilde{d}}^{6i^*}\Gamma_d^{32}, \Gamma_{\tilde{d}}^{6i^*}\Gamma_d^{33}\right)\right) \\
W^{\bar{D}} &= \left(\left(\Gamma_{\tilde{d}}^{5i}\Gamma_{\bar{d}}^{11}, \Gamma_{\tilde{d}}^{5i}\Gamma_{\bar{d}}^{12}\right), \left(\Gamma_{\tilde{d}}^{7i}\Gamma_{\bar{d}}^{21}, \Gamma_{\tilde{d}}^{7i}\Gamma_{\bar{d}}^{22}\right)\right),
\end{aligned}
\tag{D.2}
$$

where in V_{ijk}^D and $W_{ijk}^{\bar{D}}$, i specifies which field is at the vertex, j labels the fermionic eigenstate, and k labels the scalar eigenstate.

The threshold corrections to the gaugino masses can thus be expressed as

$$
\Delta m_{\tilde{i}}^Q = \sum_{j=1}^{2} \frac{\alpha_i}{\pi} C_i^Q V_{jk}^Q W_k^{\bar{Q}} M_{Q'} \mathcal{J}(M_{Q'},\, m_{\tilde{q}_k}), \qquad \mathcal{J}(M,\, m) = \frac{m^2}{M^2 - m^2} \log \frac{m^2}{M^2},
\tag{D.3}
$$

with $M_{Q'}$ the physical mass of the heavy messenger. There is an analogous expression for U, while

$$
\Delta m_{\tilde{i}}^D = \sum_{i=1}^{3} \sum_{l=1}^{2} \frac{\alpha_i}{\pi} C_i^D V_{ijk}^D W_{ljk}^{\bar{D}} M_{D'}^j \mathcal{J}(M_{D'}^j,\, m_{\tilde{d}_k}).
\tag{D.4}
$$

The Dynkin indices weighted by degrees of freedom C_i^M are given by

$$
\begin{aligned}
C^Q &= (1/6, 3/2, 1) \\
C^U &= (4/3, 0, 1/2) \\
C^D &= (1/3, 0, 1/2)
\end{aligned}
\tag{D.5}
$$

for $(U(1)_Y, SU(2)_L, SU(3)_C)$. Our extra matter was introduced in complete representations of $SU(5)$, so we still need to account for E, l_4, and L. Since our flavor model does not discuss the leptonic sector in any detail, here we simply assume that the parameters for E are the same as those for U, and identify l_4 with d_4, as well as L

with D. Although they live in the same GUT multiplets, the C factors are of course different:

$$\begin{aligned} C^E &= (1,0,0) \\ C^L &= (1/2,1/2,0). \end{aligned} \tag{D.6}$$

Appendix E

Formulae for Select Flavor Observables

1. B_s mixing: Wilson Coefficient of $\mathcal{O}_1$:

In addition to the usual box diagram MSSM contributions to meson mixing involving squarks and gluinos, our model gives a contribution coming from analogous box diagrams with flavons/messengers and flavinos/smessengers. Since the 1$^{\text{st}}$ generation fermions do not couple to the flavon sector, this additional contribution only exists for mesons that do not contain 1$^{\text{st}}$ generation quarks. The most precisely measured of these is B_s. We therefore calculate the box diagram for B_s mixing and extract the coefficient of the effective operator $\mathcal{O}_1$, given in Eq. (4.36), which we then use to calculate the contribution to the mass splitting and CP-violating phase in the B_s system.

a. Flavino:

$$C_1 = \frac{c_q^2}{128\pi^2} \widetilde{F}_{2i}^{q^*} \widetilde{F}_{3i}^{q} \widetilde{F}_{2j}^{q^*} \widetilde{F}_{3j}^{q} \Gamma_{\tilde{q}}^{5k} \Gamma_{\tilde{q}}^{5k^*} \Gamma_{\tilde{q}}^{5l} \Gamma_{\tilde{q}}^{5l^*} \mathcal{B}_1(m_{\tilde{\phi}_i},\, m_{\tilde{\phi}_j},\, m_{\tilde{q}_k},\, m_{\tilde{q}_l}), \tag{E.1}$$

$$\begin{aligned}
\mathcal{B}_1(M_1,\, M_2,\, m_1,\, m_2) \;\; = \;\; & \frac{1}{(m_1^2 - m_2^2)(m_1^2 - M_1^2)(m_1^2 - M_2^2)(m_2^2 - M_1^2)(m_2^2 - M_2^2)(M_1^2 - M_2^2)} \\
& \times \Bigg(m_1^4 m_2^4 (M_1^2 - M_2^2) \log \frac{m_1^2}{m_2^2} + m_1^4 M_1^4 (M_2^2 - m_2^2) \log \frac{m_1^2}{M_1^2} \\
& + m_1^4 M_2^4 (m_2^2 - M_1^2) \log \frac{m_1^2}{M_2^2} + m_2^4 M_1^4 (m_1^2 - M_2^2) \log \frac{m_2^2}{M_1^2} \\
& + m_2^4 M_2^4 (M_1^2 - m_1^2) \log \frac{m_2^2}{M_2^2} + M_1^4 M_2^4 (m_1^2 - m_2^2) \log \frac{M_1^2}{M_2^2} \Bigg) .
\end{aligned} \tag{E.2}$$

b. Flavon:

$$\begin{aligned}
C_1 \;\; = \;\; & \frac{c_q^2}{512\pi^2} \left(\widehat{F}_{2i}^{(q)Re} + i \widehat{F}_{2i}^{(q)Im} \right)^* \left(\widehat{F}_{3i}^{(q)Re} + i \widehat{F}_{3i}^{(q)Im} \right) \\
& \times \left(\widehat{F}_{2j}^{(q)Re} + i \widehat{F}_{2j}^{(q)Im} \right)^* \left(\widehat{F}_{3j}^{(q)Re} + i \widehat{F}_{3j}^{(q)Im} \right) \mathcal{B}_2(m_{\phi_i},\, m_{\phi_j},\, M_{Q'}),
\end{aligned} \tag{E.3}$$

$$\mathcal{B}_2(m_1,\, m_2,\, M) = \frac{m_1^4 \log \frac{m_1^2}{M^2}}{(M^2 - m_1^2)^2 (m_1^2 - m_2^2)} - \frac{m_2^4 \log \frac{m_2^2}{M^2}}{(M^2 - m_2^2)^2 (m_1^2 - m_2^2)} + \frac{M^2}{(M^2 - m_1^2)(M^2 - m_2^2)}, \tag{E.4}$$

where the total Wilson coefficient for $\mathcal{O}_1$ is the sum of Eqs. (E.1) and (E.3).

2. CEDM:

a. Flavino:

Our model gives additional contributions to chromo-EDMs for 2nd and 3rd generation quarks. The diagrams involve flavinos and flavons (*cf.* Fig. 4.13) and are constructed from the ones shown in Fig. 4.4 by attaching a gluon line to a member of the messenger multiplet. The flavino loops are typically the dominant contribution, and they give:

$$\tilde{d}_{in} = \frac{v \sin\beta \, m_{\tilde{\phi}_j}}{16\pi^2} r_i^q r_n^u Im\{\widetilde{F}^q_{ij} \widetilde{F}^u_{nj} \Gamma^{5k^*}_{\tilde{q}} \Gamma^{5l^*}_{\tilde{u}} H^{QU}_{kl}\} \Big(\mathcal{D}(m_{\tilde{\phi}_j},\, m_{\tilde{q}_k},\, m_{\tilde{u}_l}) + \mathcal{D}(m_{\tilde{\phi}_j},\, m_{\tilde{u}_l},\, m_{\tilde{q}_k})\Big)\,, \tag{E.5}$$

where $v = 174$ GeV is the Higgs VEV; the first term corresponds to gluon emission from the $\tilde{q}$ and the second from $\tilde{u}$, and

$$\mathcal{D}(M,\, \mu,\, m) = \frac{1}{2M^4} \int_0^1 dw\, w \int_0^{1-w} dx \int_0^{1-x-w} dy \left[w + (x+y)\frac{\mu^2}{M^2} + (1-w-x-y)\frac{m^2}{M^2} \right]^{-2}. \tag{E.6}$$

b. Flavon:

We also get loops with flavons and smessengers. Just as for the loops generating Yukawa couplings, there are contributions with and without mass insertions on the messenger lines. For the dipole calculation though, the latter are manifestly finite. Those without mass insertions are typically larger, giving

$$\begin{aligned} \tilde{d}_{ij} \;\; &= \;\; -\frac{v\cos\beta}{32\pi^2} r_i^q r_j^u Im\left\{ \bar{\lambda}^*_U \left(\widehat{F}^{(q)Re}_{ik} + i\widehat{F}^{(q)Im}_{ik}\right)\left(\widehat{F}^{(u)Re}_{jk} + i\widehat{F}^{(u)Im}_{jk}\right)\right\} \\ &\times\big(\mathcal{D}^{\rm no-MI}_{\phi}(M_{Q'},\, M_{U'},\, m_{\phi_k}) + \mathcal{D}^{\rm no-MI}_{\phi}(M_{U'},\, M_{Q'},\, m_{\phi_k})\big), \end{aligned} \tag{E.7}$$

where

$$\mathcal{D}_\phi^{\mathrm{no-MI}}(M,\,\mu,\,m) = \frac{1}{2\,m^2}\int_0^1 dw \int_0^{1-w} dy \int_0^{1-y-w} dz \times\left(\frac{3(y+z)-1}{[(1-w-y-z)+(w+z)M^2/m^2+y\,\mu^2/m^2]} - \frac{wM^2}{m^2\,[(1-w-y-z)+(w+z)M^2/m^2+y\,\mu^2/m^2]^2}\right). \tag{E.8}$$

Lastly, we come to the flavon loop with mass insertions:

$$\tilde{d}_{ij} = \frac{v\sin\beta\,M_{Q'}M_{U'}}{32\pi^2} c_u c_q\, r_i^q r_j^u Im\left\{\lambda_U\left(\widehat{F}_{ik}^{(q)Re}+i\widehat{F}_{ik}^{(q)Im}\right)\left(\widehat{F}_{jk}^{(u)Re}+i\widehat{F}_{jk}^{(u)Im}\right)\right\} \times\big(\mathcal{D}_\phi^{\mathrm{MI}}(M_{Q'},\,M_{U'},\,m_{\phi_k})+\mathcal{D}_\phi^{\mathrm{MI}}(M_{U'},\,M_{Q'},\,m_{\phi_k})\big), \tag{E.9}$$

with

$$\mathcal{D}_\phi^{\mathrm{MI}}(M,\,\mu,\,m) = \frac{1}{2\,m^4}\int_0^1 dw \int_0^{1-w} dy \int_0^{1-y-w} dz \times\frac{(w+y+z)}{[(1-w-y-z)+(w+z)M^2/m^2+y\,\mu^2/m^2]^2}. \tag{E.10}$$

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Vita

Thomas Zorawski was born in Łódź, Poland and grew up in New York City. He was a Presidential Honors Scholar at New York University, where he graduated *summa cum laude* and Phi Beta Kappa in 2007 with a B.A. in physics. He enrolled in the Physics PhD program at Johns Hopkins University that fall. His research has focused on particle physics beyond the Standard Model, mainly supersymmetric model-building. In 2012, he received the *Rowland Prize for Innovation and Excellence in Teaching.* He is leaving academic physics (at least for now), but will continue to follow the latest developments with great interest.